TM-9 part 1

Determine laws of motion of bodies 2 and 3 (fig. 1), absolute velocity and acceleration of point M. It is known, that the law of motion of body 1 is $x_1 = 5t^2$, m; external radius of body 2 is $R_2 = 1$ m, internal radius of body 2 is $r_2 = 0.5$ m, and a radius of body 3 is $R_3 = 0.75$ m, time parameter is $t = 1$ s.

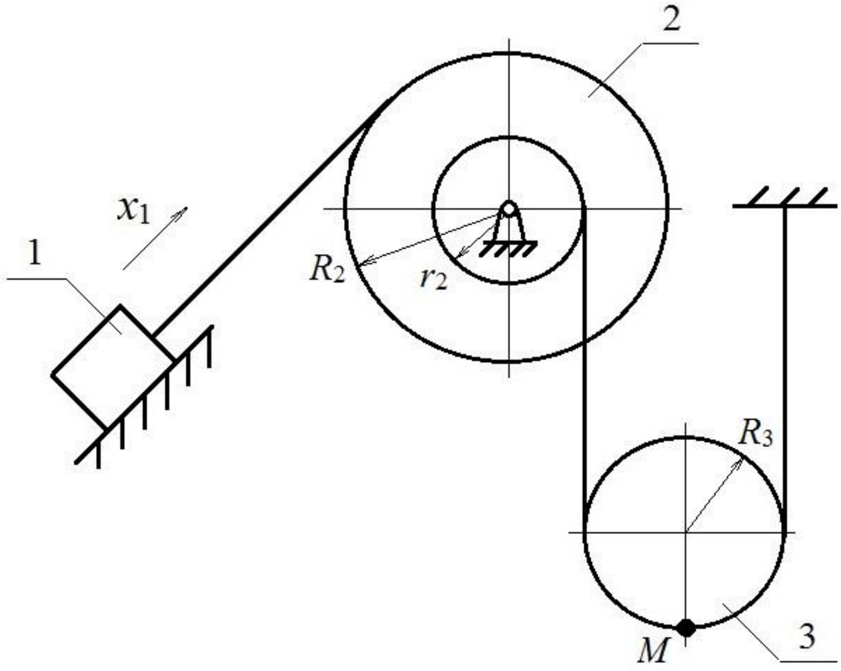

Fig. 1 Initial scheme

Solution

Given:
$x_1 = 5t^2$, m
$R_2 = 1$ m
$r_2 = 0.5$ m
$R_3 = 0.75$ m
$t = 1$ s
Find:
φ_2, φ_3, v_M, a_M

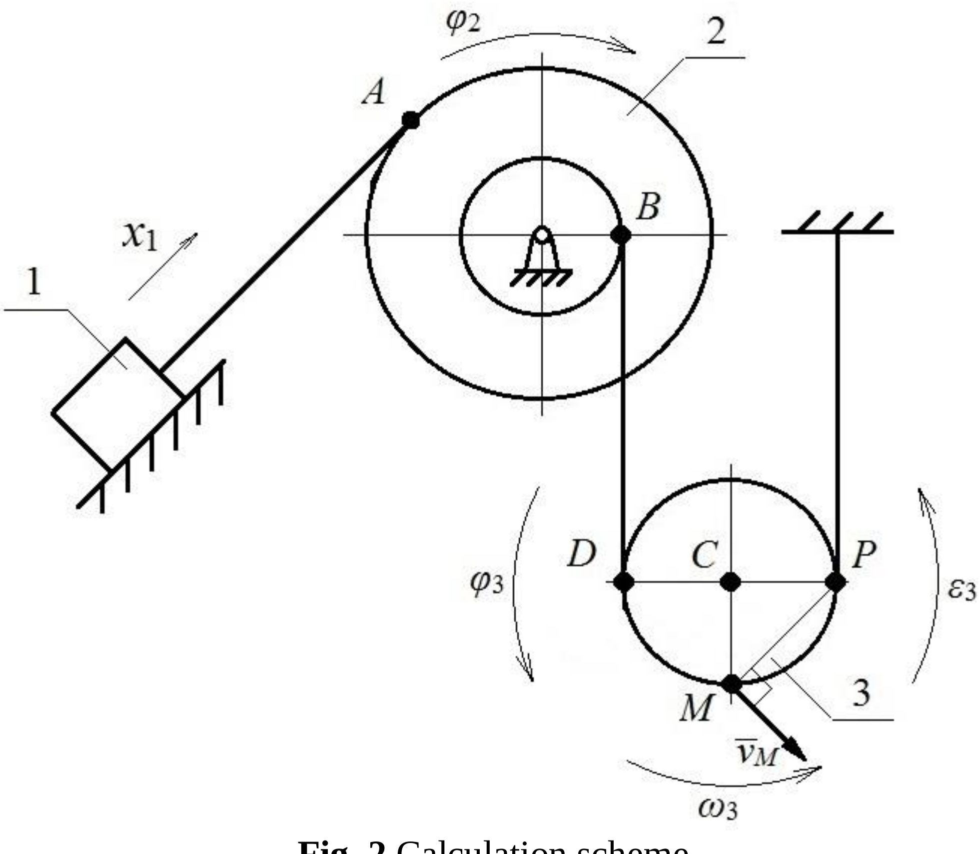

Fig. 2 Calculation scheme

Construct a calculation scheme (fig. 2). Place additional points *A*, *B*, *C*, *D*, *P* on it, and directions of motion of bodies 2 and 3, directions of angular velocity ω_3 and angular acceleration ε_3 of body 3, and a vector of absolute velocity v_M.

Body 1 moves linearly, body 2 rotates, and body 3 performs plane motion. Body 1 and point *A* are connected through an ideal string (the one that doesn't deform). Points *B* and *D*, *P* and a support are also connected through an ideal string.

Direction of motion of body 1 is given in the initial data. If body 1 moves linearly along an inclined surface and is connected with point *A* through an ideal string, then body 2 rotates clockwise (designated as φ_2 in fig. 2). Direction of motion of body 3 can be determined analogically. Body 3 rotates counterclockwise (designated as φ_3 in fig. 2).

If a string is ideal, then movement of body 1 is equal to a length of a string, which is wound up on external radius of body 2. Analogically, a length of a string, which is unwound from internal radius of body 2, is equal to a length of a string, which is wound up on radius of body 3. Express string lengths using the angle and radius of rotation.

Determine the equations of kinematic connections

$$x_1 = \varphi_2 \cdot R_2,$$

$$\varphi_2 \cdot r_2 = \varphi_3 \cdot 2R_3.$$

From where the laws of motion of bodies 2 and 3 are

$$\varphi_2 = \frac{x_1}{R_2} = \frac{5t^2}{1} = 5t^2, rad,$$

$$\varphi_3 = \frac{\varphi_2 \cdot r_2}{2R_3} = \frac{5t^2 \cdot 0.5}{2 \cdot 0.75} = 1.67t^2, rad.$$

Determine angular velocity ω_3 and angular acceleration ε_3 of body 3.

$$\omega_3 = \frac{d\varphi_3}{dt} = \frac{d(1.67t^2)}{dt} = 3.34t, rad/s,$$

$$\varepsilon_3 = \frac{d\varepsilon_3}{dt} = \frac{d(3.34t)}{dt} = 3.34, rad/s^2.$$

ω_3 and ε_3 are directed counterclockwise since they are positive for a positive *t*.

Body 3 performs plane motion and is connected with a support through a string. Then a center of rotation (instantaneous center of velocity) of body 3 is point *P*.

Determine absolute velocity of point *M*.

$$v_M = \omega_3 \cdot MP = 3.34t \cdot \sqrt{2} \cdot R_3 = 3.53t, m/s.$$

At the moment of time $t = 1$ s

$$v_M = 3.53 \cdot 1 = 3.53\, m/s.$$

Absolute acceleration of a body, which performs plane motion, is determined as a vector sum of accelerations of a pole $\overline{a_C}$, normal $\overline{a_{MC}^n}$, and tangent $\overline{a_{MC}^\tau}$ accelerations of the considered point around this pole. The pole is the point, acceleration of which is known or is easy to calculate. Define point *C* as a pole since it is a center of mass of body 3 and moves linearly.

$$\overline{a_M} = \overline{a_C} + \overline{a_{MC}^n} + \overline{a_{MC}^\tau}$$

Determine acceleration of point *C*

$$x_C = \varphi_3 \cdot CP = 1.67t^2 \cdot R_3 = 1.25t^2, m,$$

$$v_C = \frac{dx_C}{dt} = \frac{d(1.25t^2)}{dt} = 2.5t, m/s,$$
$$a_C = \frac{dv_C}{dt} = \frac{d(2.5t)}{dt} = 2.5\, m/s^2.$$

Determine normal and tangent accelerations of point M relatively to the pole C at the moment of time $t = 1$ s

$$a_{MC}^n = \omega_3^2 \cdot MC = (3.34t)^2 \cdot R_3 = 8.37\ (m/s^2),$$
$$a_{MC}^\tau = \varepsilon_3 \cdot MC = 3.34 \cdot R_3 = 2.51\ (m/s^2).$$

In order to determine absolute acceleration of point M, project the vectors of determined accelerations on coordinate axes x and y (fig. 3). Note that normal acceleration $\overline{a_{MC}^n}$ is directed from point M to the pole C, and tangent acceleration $\overline{a_{MC}^\tau}$ is perpendicular to the normal acceleration and is directed along angular acceleration ε_3.

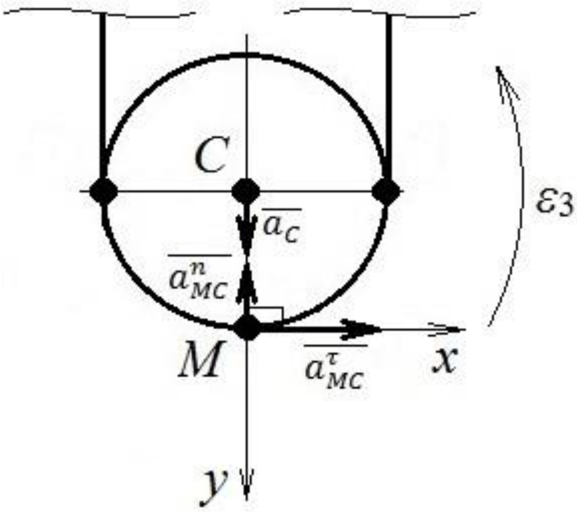

Fig. 3 Acceleration calculation scheme

Then absolute acceleration a_M of point M

$$a_M = \sqrt{\left(\sum a_{ix}\right)^2 + \left(\sum a_{iy}\right)^2} = \sqrt{(a_{MC}^\tau)^2 + (a_C - a_{MC}^n)^2} =$$
$$= \sqrt{2.51^2 + (2.5 - 8.37)^2} = 6.38\ (m/s^2).$$

Answer: $\varphi_2 = 5t^2, rad$, $\varphi_3 = 1.67t^2, rad$, $v_M = 3.53\, m/s$, $a_M = 6.38\, m/s^2$.