**ТМ-9 part 1**

Determine laws of motion of bodies 2 and 3 (fig. 1), absolute velocity and acceleration of point *М*. It is known, that the law of motion of body 1 is *x*1 = 5*t*2, m; external radius of body 2 is *R*2 = 1 m, internal radius of body 2 is *r*2 = 0.5 m, and a radius of body 3 is *R*3 = 0.75 m, time parameter is *t* = 1 s.



**Fig. 1** Initial scheme

**Solution**

|  |  |
| --- | --- |
| Given: *x*1 = 5*t*2, m*R*2 = 1 m*r*2 = 0.5 m*R*3 = 0.75 m*t* = 1 sFind:*φ*2, *φ*3, *vM*, *aM* | E:\Онищенко\ТМ-9ч1.розр.сх2.jpg |
|  | **Fig. 2** Calculation scheme |

Construct a calculation scheme (fig. 2). Place additional points *A*, *B*, *C*, *D*, *P* on it, and directions of motion of bodies 2 and 3, directions of angular velocity *ω*3 and angular acceleration *ε*3 of body 3, and a vector of absolute velocity *vM*.

Body 1 moves linearly, body 2 rotates, and body 3 performs plane motion. Body 1 and point *A* are connected through an ideal string (the one that doesn’t deform). Points *B* and *D*, *P* and a support are also connected through an ideal string.

Direction of motion of body 1 is given in the initial data. If body 1 moves linearly along an inclined surface and is connected with point *A* through an ideal string, then body 2 rotates clockwise (designated as *φ*2 in fig. 2). Direction of motion of body 3 can be determined analogically. Body 3 rotates counterclockwise (designated as *φ*3 in fig. 2).

If a string is ideal, then movement of body 1 is equal to a length of a string, which is wound up on external radius of body 2. Analogically, a length of a string, which is unwound from internal radius of body 2, is equal to a length of a string, which is wound up on radius of body 3. Express string lengths using the angle and radius of rotation.

Determine the equations of kinematic connections

$x\_{1}=φ\_{2}∙R\_{2}$,

$φ\_{2}∙r\_{2}=φ\_{3}∙2R\_{3}$.

From where the laws of motion of bodies 2 and 3 are

$φ\_{2}=\frac{x\_{1}}{R\_{2}}=\frac{5t^{2}}{1}=5t^{2}, rad$,

$φ\_{3}=\frac{φ\_{2}∙r\_{2}}{2R\_{3}}=\frac{5t^{2}∙0.5}{2∙0.75}=1.67t^{2}, rad$.

Determine angular velocity *ω*3 and angular acceleration *ε*3 of body 3.

$ω\_{3}=\frac{dφ\_{3}}{dt}=\frac{d(1.67t^{2})}{dt}=3.34t, rad/s$,

$ε\_{3}=\frac{dε\_{3}}{dt}=\frac{d(3.34t)}{dt}=3.34, rad/s^{2}$.

*ω*3 and *ε*3 are directed counterclockwise since they are positive for a positive *t*.

Body 3 performs plane motion and is connected with a support through a string. Then a center of rotation (instantaneous center of velocity) of body 3 is point *Р*.

Determine absolute velocity of point *М*.

$v\_{M}=ω\_{3}∙MP=3.34t∙\sqrt{2}∙R\_{3}=3.53t, m/s$.

At the moment of time *t* = 1 s

$v\_{M}=3.53∙1=3.53 m/s$.

Absolute acceleration of a body, which performs plane motion, is determined as a vector sum of accelerations of a pole $\overline{a\_{C}}$, normal $\overline{a\_{MC}^{n}}$, and tangent $\overline{a\_{MC}^{τ}}$ accelerations of the considered point around this pole. The pole is the point, acceleration of which is known or is easy to calculate. Define point *С* as a pole since it is a center of mass of body 3 and moves linearly.

$$\overline{a\_{M}}=\overline{a\_{C}}+\overline{a\_{MC}^{n}}+\overline{a\_{MC}^{τ}}$$

Determine acceleration of point *С*

$x\_{C}=φ\_{3}∙CP=1.67t^{2}∙R\_{3}=1.25t^{2}, m$*,*

$v\_{C}=\frac{dx\_{C}}{dt}=\frac{d(1.25t^{2})}{dt}=2.5t, m/s$*,*

$a\_{C}=\frac{dv\_{C}}{dt}=\frac{d(2.5t)}{dt}=2.5 m/s^{2}$.

Determine normal and tangent accelerations of point *М* relatively to the pole *С* at the moment of time *t* = 1 s

$a\_{MC}^{n}=ω\_{3}^{2}∙MC=\left(3.34t\right)^{2}∙R\_{3}=8.37 (m/s^{2})$,

$a\_{MC}^{τ}=ε\_{3}∙MC=3.34∙R\_{3}=2.51 (m/s^{2})$.

In order to determine absolute acceleration of point *М*, project the vectors of determined accelerations on coordinate axes *x* and *y* (fig. 3). Note that normal acceleration $\overline{a\_{MC}^{n}}$ is directed from point *M* to the pole *C*, and tangent acceleration $\overline{a\_{MC}^{τ}}$ is perpendicular to the normal acceleration and is directed along angular acceleration $ε\_{3}$.



**Fig. 3** Acceleration calculation scheme

Then absolute acceleration *aM* of point *М*

$$a\_{M}=\sqrt{\left(\sum\_{}^{}a\_{ix}\right)^{2}+\left(\sum\_{}^{}a\_{iy}\right)^{2}}=\sqrt{\left(a\_{MC}^{τ}\right)^{2}+\left(a\_{C}-a\_{MC}^{n}\right)^{2}}==\sqrt{2.51^{2}+\left(2.5-8.37\right)^{2}}=6.38 (m/s^{2}).$$

**Answer:** $φ\_{2}=5t^{2}, rad$, $φ\_{3}=1.67t^{2}, rad$, $v\_{M}=3.53 m/s$, $a\_{M}=6.38 m/s^{2}$.