2. STRUCTURAL ANALYSE AND SYNTHESIS OF MECHANISMS

2.1 Link

To a beginner, for short, the term *machine* may be defined as a device which receives energy in some available form and uses it to do certain particular kind of work. *Mechanism* may be defined as a contrivance which transforms motion from one form to another.

A machine consists of a number of parts or bodies. In this chapter, we shall study the mechanisms of the various parts or bodies from which the machine is assembled. This is done by making one of the parts as fixed, and the relative motion of other parts is determined with respect to the fixed part.

Each part of a machine, which moves relative to some other part, is known as a *kinematic link* (or simply link). A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another. Even if two or

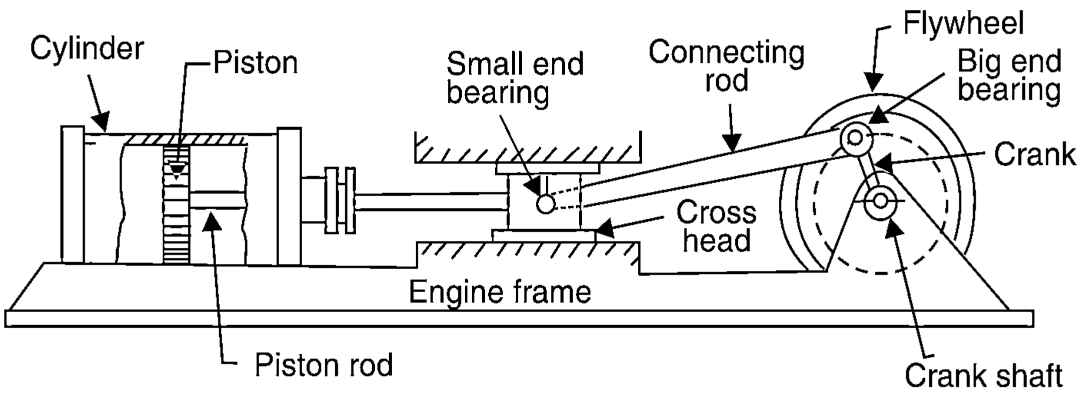


Fig.2.1. Reciprocating steam engine

more connected parts are manufactured separately, they cannot be treated as different links unless there is a relative motion between them. For example, in a reciprocating steam engine, as shown in Fig. 2.1, piston, piston rod and crosshead constitute one link; connecting rod with big and small end bearings constitute a second link; crank,

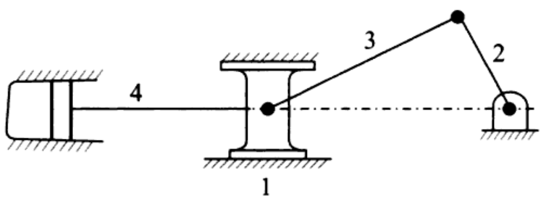


Fig. 2.2. Steam engine mechanism

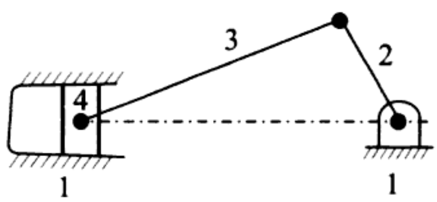


Fig. 2.3. I.C. engine mechanism

crank shaft and flywheel third link and the cylinder, engine frame and main bearings fourth link. Therefore, slider-crank mechanisms of a steam engine (Fig. 2.2) and I.C. engine (2.3) are just the same. So, a *link* may be defined as a single part (or an assembly of rigidly connected parts) of a machine, which is a resistant body having a motion relative to other parts of the machine (mechanism).

A link needs not to be rigid body, but it must be a *resistant body*. A body is said to be a resistant one if it is capable of transmitting the required forces with negligible deformation. Based on above considerations a spring which has no effect on the kinematics of a device and has significant deformation in the direction of applied force is not treated as a link but only as a device to apply force (Fig.2.4).They are usually ignored during kinematic analysis, and their "force-effects" are introduced during dynamic analysis.

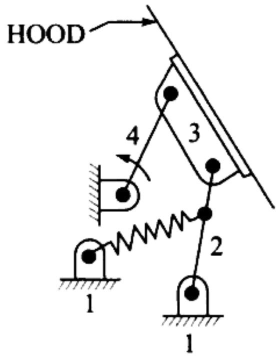


Fig. 2.4. Four bar automobile-hood mechanism

There are machine members which possess one-way rigidity. For instance, because of their resistance to deformation under tensile load, belts (Fig. 2.5), ropes and chains are treated as links only when they are in tension. Similarly, liquids on account of their incompressibility can be treated as links only when transmitting compressive force.

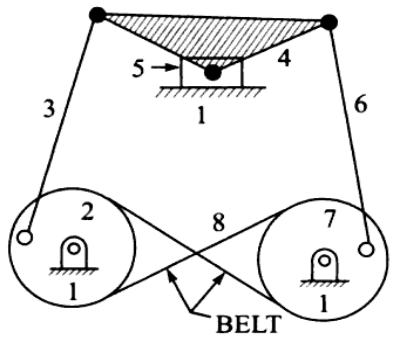


Fig. 2.5. Mechanism with belt-pulley combination

Thus a link should have the following two characteristics:
1. It should have relative motion, and
2. It must be a resistant body.

Structure is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure. The following differences between a machine (mechanism) and a structure are important from the subject point of view: 1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another. 2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work. 3. The links of a machine may transmit both power and motion, while the members of a structure transmit forces only.

The kind of relative motion between links of a mechanism is controlled by the form of the contacting surfaces of the adjacent (connected) links. These contacting surfaces may be thought of as 'working surfaces' of the connection between adjacent links. For instance, the connection between a lathe carriage and its bed is through working surfaces (ways) which are so shaped that only motion of translation is possible. Similarly, the working surface of I.C. engine piston and connecting rod at piston pin are so shaped that relative motion of rotation alone is possible. Each of these working surfaces is called an *element*.

An element may therefore be defined as a geometrical form provided on a link so as to ensure a working surface that permits desired relative motion between connected links.

2.2 Classification of Links

A link can be called singular (unitary), binary, ternary, quaternary (etc.) link depending on the number of elements it has for pairing with other links. Thus a link carrying a single element is called a singular (unitary) link and a link with two elements is called a binary link. Similarly, a link having three elements is called a ternary link while a link having four elements is called a quaternary link. These links, along with their convention representation, are shown in Fig. 2.6.

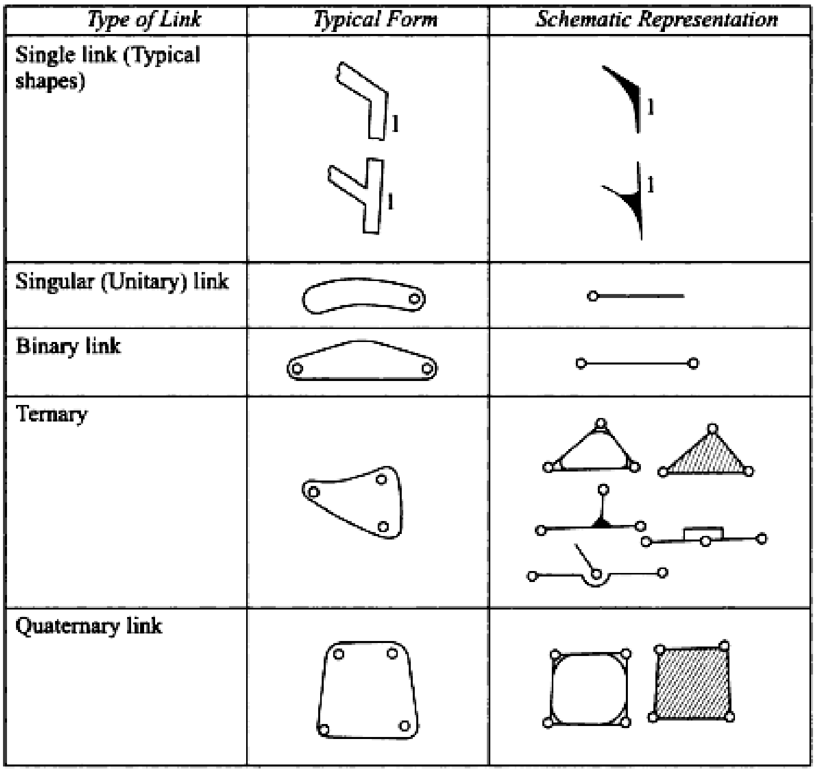


Fig. 2.6. Conventional representation of different types of links

2.3 Kinematic Pair

The two contacting elements of a connection constitute a kinematic pair. A pair may also be defined as a connection between two adjacent links that permits a definite relative motion between them. It may be noted that the above statement is generally true. In the case of multiple joint, however, more than two links can be connected at a kinematic pair (also known as joint). Cylindrical contacting surfaces between I.C. engine cylinder and piston constitute a pair. Similarly, cylindrical contacting surfaces of a rotating shaft and a journal bearing also constitute a pair.

When all the points in different links in a mechanism move in planes which are mutually parallel the mechanism is said to have a planar motion. A motion other than planar motion is spatial motion.

When the links are assumed to be rigid in kinematics, there can be no change in relative positions of any two arbitrarily chosen points on the same link. In particular, relative position(s) of pairing elements on the same link does not change. As a consequence of assumption of rigidity, many of the intricate details, shape and size of the actual part (link) become unimportant in kinematic analysis. For this reason it is customary to draw highly simplified schematic diagrams which contain only the important features in respect of the shape of each link (e.g., relative locations of pairing elements). This necessarily requires to completely suppressing the information about real geometry of manufactured parts. Schematic diagrams of various links, showing relative location of pairing elements, are shown in Fig. 2.2.-2.5. Conventions followed in drawing kinematic diagram are also shown there.

In drawing a kinematic diagram, it is customary to draw the parts (links) in the most simplified form so that only those dimensions are considered which affect the relative motion. One such simplified kinematic diagram of slider-crank mechanism of an I.C. engine is shown in Fig. 2.3 in which connecting rod 3 and crank 2 are represented by lines joining their respective pairing elements. The piston has been represented by the slider 4 while cylinder (being a stationary member) has been represented by frame link 1.

It may be noted, however, that these schematics, have a limitation in that they have little resemblance to the physical hardware. And, one should remember that kinematic diagrams are particularly useful in kinematic analysis and synthesis but they have very little significance in designing the machine components of such a mechanism.

2.4 Classification of Pairs

2.4.1 Classification of Pairs Based on Type of Relative Motion

The relative motion of a point on one element relative to the other on mating element can be that of turning, sliding, screw (helical direction), planar, cylindrical or spherical. The controlling factor that determines the relative motions allowed by a given joint is the shapes of the mating surfaces or elements. Each type of joint has its own characteristic shapes for the elements, and each permits a particular type of motion, which is determined by the possible ways in which these elemental surfaces can move with respect to each other. The shapes of mating elemental surfaces restrict the totally arbitrary motion of two unconnected links to some prescribed type of relative motion.

Turning Pair (Also called a hinge, a pin joint or a revolute pair). This is the most common type of kinematic pair and is designated by the letter *R*.

A pin joint has cylindrical element surfaces and assuming that the links cannot slide axially, these surfaces permit relative motion of rotation only. A pin joint allows the two connected links to experience relative rotation about the pin centre. Thus, the pair permits only one degree of freedom. Thus, the pair at piston pin, the pair at crank pin and the pair formed by rotating crank-shaft in bearing are all example of turning pairs.

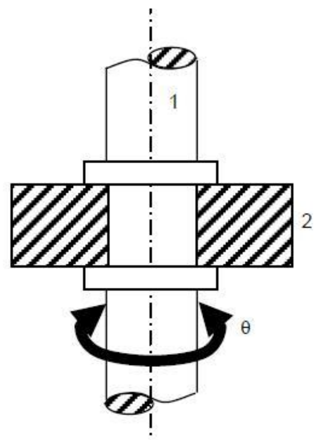


Fig. 2.7. Turning (revolute) pair *R*, *F*=1

Sliding or Prismatic Pair. This is also a common type of pair and is designated as *P* (Fig.2.8).

This type of pair permits relative motion of sliding only in one direction (along a line) and as such has only one degree of freedom. Pairs between piston and cylinder, cross-head and guides, die-block and slot of slotted lever are all examples of sliding pairs.

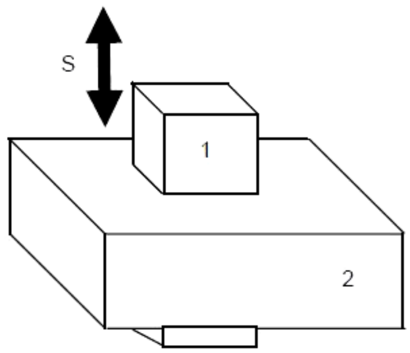


Fig. 2.8. Prismatic or sliding pair *P*, *F*=1

Screw Pair. This pair permits a relative motion between coincident points, on mating elements, along a helix curve. Both axial sliding and rotational motions are involved. But as the sliding and rotational motions are related through helix angle θ, the pair has only one degree of freedom Fig (2.9.). The pair is commonly designated by the letter *S*. Example of such pairs are to be found in translatory screws operating against rotating nuts to transmit large forces at comparatively low speed, *e.g.* in screw-jacks, screw-presses, valves and pressing screw of rolling mills. Other examples are rotating lead screws operating in nuts to transmit motion accurately as in lathes, machine tools, measuring instruments, etc.

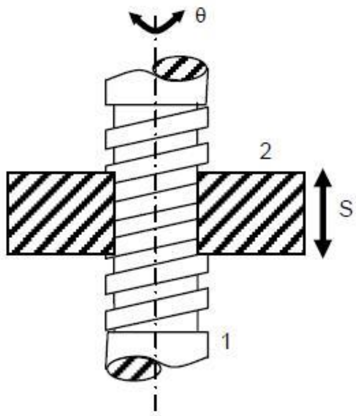


Fig. 2.9. Screw (helical) pair *S*, *F*=1

Cylindrical Pair. A cylindrical pair permits a relative motion which is a combination of rotation θ and translation *s* parallel to the axis of rotation between the contacting elements (2.10). The pair has thus two degrees of freedom and is designated by a letter *C*. A shaft free to rotate in bearing and also free to slide axially inside the bearing provides example of a cylindrical pair.

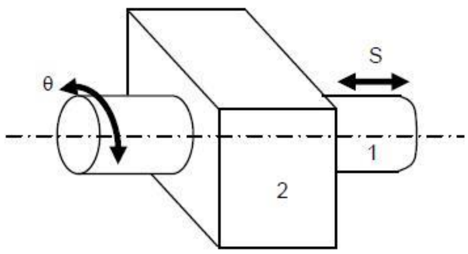


Fig. 2.10. Cylindrical *pair C*, *F*=2

Globular or Spherical Pair. Designated by the letter *G*, the pair permits relative motion such that coincident points on working surfaces of elements move along spherical surface. In other words, for a given position of spherical pair, the joint permits relative rotation about three mutually perpendicular axes. It has thus three degrees of freedom. A ball and socket joint (*e.g.*, the shoulder joint at arm-pit of a human being) is the best example of spherical pair.

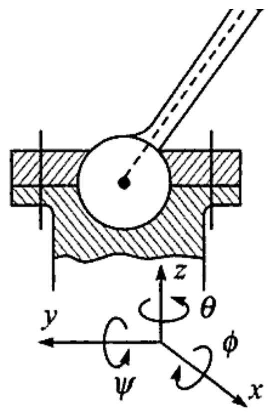


Fig. 2.11. Globular or spherical pair *G*, *F*=3

Flat pair (Planar Pair). A flat or planar pair is seldom, if ever, found in mechanisms. The pair permits a planar relative motion between contacting elements. This relative motion can be described in terms of two translatory motions in x and y directions and a rotation about third direction z, x, y, z being mutually perpendicular directions. The pair is designated as F and has three degrees of freedom.

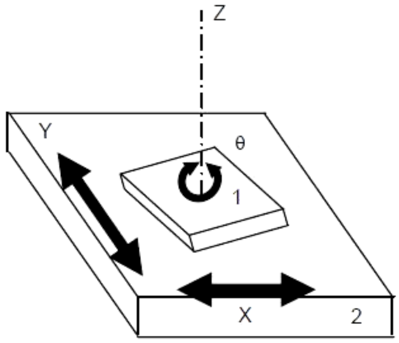


Fig. 2.12. Flat pair *F*, *F*=3

Rolling Pair. When surfaces of mating elements have a relative motion of rolling, the pair is called a rolling pair. Castor wheel of trolleys, ball and roller bearings, wheels of locomotive/wagon and rail are a few examples of this type.

2.4.2 Classification of Pairs Based on Type of Contact

This is the best known classification of kinematic pairs on the basis of nature of contact:

Lower Pair. Kinematic pairs in which there is surfaces (area) contact between the contacting elements are called lower pairs. All revolute pairs, sliding pairs, screw pairs, globular pairs, cylindrical pairs and flat pairs fall in this category.

Higher Pair. Kinematic pairs in which there is point or line contact between the contacting elements are called higher pairs. Meshing gear-teeth, cam follower pair, wheel rolling on a surface, ball and roller bearings and pawl and ratchet are a few examples of higher pairs.

Since lower pairs involve surface contact rather than line or point contact, it follows that lower pairs can be more heavily loaded for the same unit pressure. They are considerably more wear-resistant. For this reason, development in kinematics has involved more and more number of lower pairs. As against this, use of higher pairs implies lesser friction.

The real concept of lower pairs lies in the particular kind of relative motion permitted by the connected links. For instance, let us assume that two mating elements P and Q form kinematic pair. If the path traced by any point on the element P, relative to element Q, is identical to the path traced by a corresponding (coincident) point in the element Q relative to element P, then the two elements P and Q are said to form a lower pair. Elements not satisfying the above condition obviously form the higher pairs

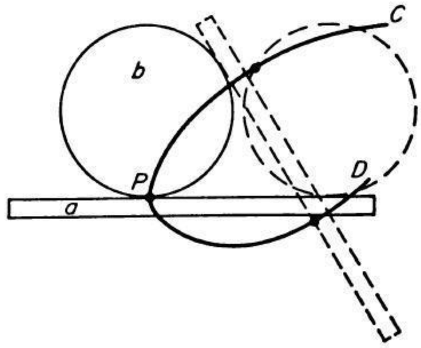


Fig. 2.13. Different paths of point *P* (*PC*-cycloid, *PD*-involute)

Since a turning pair involves relative motion of rotation about pin-axis, coincident points on the two contacting elements will have circular areas of same radius as their path. Similarly elements of sliding pair will have straight lines as the path for coincident points. In the case of screw pair, the coincident points on mating elements will have relative motion along helices. As against this a point on periphery of a disk rolling along a straight line generates cycloidal path, but the coincident point on straight line generates involute path when the straight line rolls over the disk (Fig. 2.13). The two paths are thus different and the pair is a higher pair. *As a direct sequel to the above consideration, unlike a lower pair, a higher pair cannot be inverted. That is, the two elements of the pair cannot be interchanged with each other without affecting the overall motion of the mechanism.*

Lower pairs are further subdivided into linear motion and surface motion pairs. The distinction between these two sub-categories is based on the number of degrees of freedom of the pair. Linear motion lower pairs are those having one degree of freedom, i.e. each point on one element of the pair can move only along a single line or curve relative to the other element. This category includes turning pairs, prismatic pairs and screw pairs.

Surfaces-motion lower pairs have two or more degrees of freedom. This category includes cylindrical pair, spherical pair and the planar (flat) pair.

2.4.3. Classification of Pairs Based on Degrees of Freedom

A free body in space has six degrees of freedom (d.o.f.=F=6). In forming a kinematic pair, one or more degrees of freedom are lost. The remaining degrees of freedom of the pair can then be used to classify pairs. Thus,

d.o.f. of a pair = 6 – (Number of restrains).

Tab 2.1. Classification of pairs

No in Fig.2.6	Geometrical shapes of elements in contact	Number of Restraints on		Total Number of Restraints	Class of pair
		Translatory motion	Rotary motion		
(a)	Sphere and plane	1	0	1	I
(b)	Sphere inside a cylinder	2	0	2	II
(c)	Cylinder on plane	1	1	2	II
(d)	Sphere in spherical socket	3	0	3	III
(e)	Sphere in slotted cylinder	2	1	3	III
(f)	Prism on a plane	1	2	3	III
(g)	Spherical ball in slotted socket	3	1	4	VI
(h)	Cylinder in cylindrical hollow	2	2	4	IV
(i)	Collared cylinder in hollow cylinder	3	2	5	V
(j)	Prism in prismatic hollow	2	3	5	V

A kinematic pair can therefore be classified on the basis of number of restrains imposed on the relative motion of connected links. This is done in Tab. 2.1 for different forms of pairing element shown in Fig. 2.14.

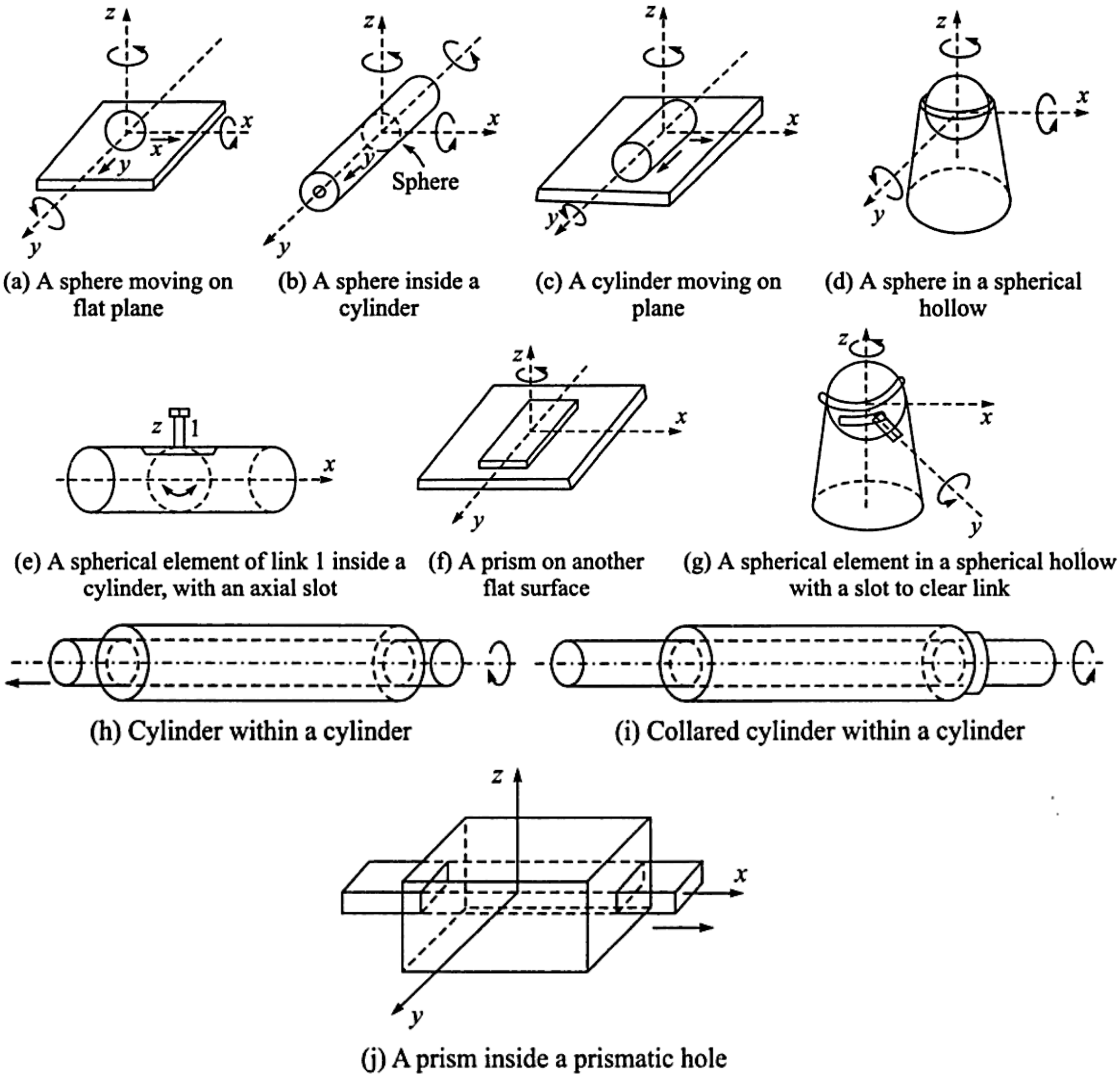


Fig. 2.14. Classification of pairs based on degrees of freedom

2.4.4. Classification of Pairs Based on Type of Closure

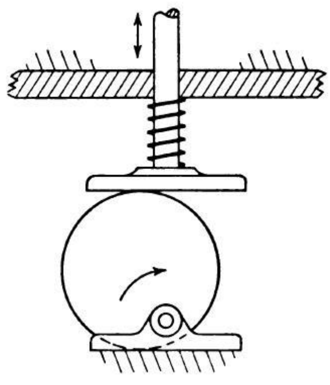


Fig. 2.15. Cam and roller-follower

Another important way of classifying pairs is to group them as closed or self closed kinematic pairs and open kinematic pairs.

In closed pairs, one element completely surrounds the other so that it is held in place in all possible positions. Restraint is achieved only by the form of pair and, therefore, the pair is called closed or self-closed pair. Therefore, closed pairs are those pairs in which elements are held together mechanically. All the lower pairs and a few higher pairs fall in the category of closed pairs

As against this, open kinematic pairs maintain relative positions only when there is some external means to prevent separation of contacting elements. Open pairs are also sometimes called as unclosed pairs. A cam and roller-follower mechanism, held in contact due to spring and gravity force, is an example of this type (Fig. 2.15).

2.5. Kinematic Chain.

A kinematic chain can be defined as an assemblage of links which are inter-connected through pairs, permitting relative motion between links. A chain is called a closed chain when links are so connected in sequence that first link is connected to the last, ensuring that all pairs are complete because of mated elements forming working surfaces at joints. As against this, when links are connected in sequence, with first link not connected to the last (leaving incomplete pairs), the chain is called an open chain. Examples of planar open loop chain are not many but they have many applications in the area of robotics and manipulators as space mechanisms. An example of a planar open-loop chain, which permits the use of a singular link (a link with only one element on it), is the common weighing scale shown in Fig. 2.16.

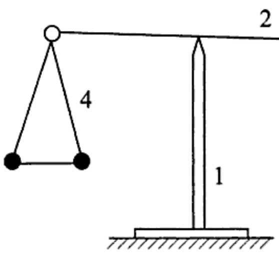


Fig. 2.16. Weighing scale

Various links are numbered in the figure. Links 3, 1 and 4 are singular links.

From the subject point of view, a mechanism may now be defined as a movable closed kinematic chain with one of its links fixed.

A mechanism with four links is known as *simple mechanism*, and the mechanism with more than four links is known as *compound mechanism*. Let's repeat once again when a mechanism is required to transmit power or to do some particular type of work, it then becomes a *machine*. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely. A little consideration will show that a mechanism may be regarded as a machine in which each part is reduced to the simplest form to transmit the required motion.

Sometimes one prefers to reserve the term *linkage* to describe mechanisms consisting of lower pairs only. But on a number of occasions this term has been used rather loosely synonymous to the term mechanism.

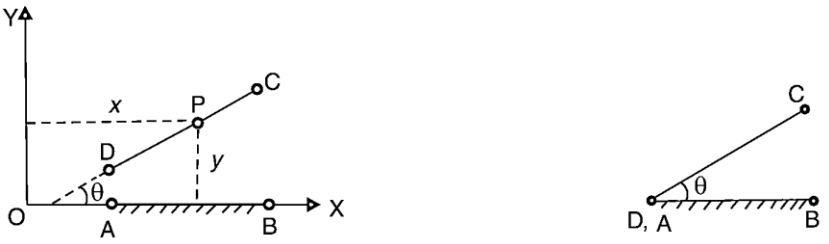
2.6. Number of Degrees of Freedom of Mechanisms

Constrained motion is defined as that motion in which all points move in predetermined paths, irrespective of the directions and magnitudes of the applied forces. Mechanisms may be categorized in number of ways to emphasize their similarities and differences. One such grouping can be to divide mechanisms into planar, spherical and spatial categories. As seen earlier, a planar mechanism is one in which all particles on any link of a mechanism describe plane curves in space and all these curves lie in parallel planes.

In the design and analysis of a mechanism, one of the most important concerns is the number of degrees of freedom, also called mobility, of the mechanism.

The number of independent input parameters which must be controlled independently so that a mechanism fulfills its useful engineering purpose is called its degree of freedom or mobility. Degree of freedom equal to 1 (d.o.f. = F=1) implies that when any point on the mechanism is moved in a prescribed way, all other points have uniquely determined (constrained) motions. When d.o.f. =2, it follows that two independent motions must be introduced at two different points in a mechanism, or two different forces or moments must be present as output resistances (as is the case in automotive differential

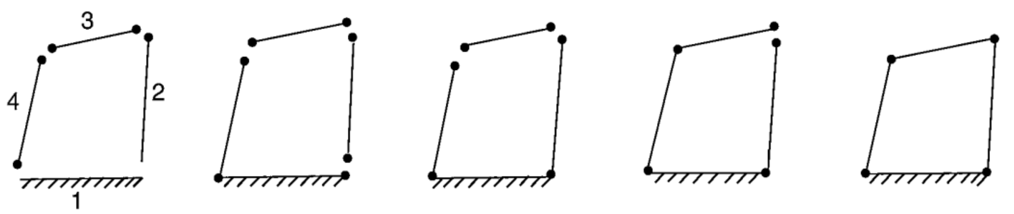
It is possible to determine the number of degrees of freedom of a mechanism directly from the number of links and the number and types of pairs which it includes. In order to develop the relationship in general, consider two links AB and CD in a plane motion as shown in Fig. 2.17 (a) The link AB with co-ordinate system OXY is taken as the reference link (or fixed link). The position of point P on the moving link CD can be completely specified by the three variables, *i.e.* the co-ordinates of the point P denoted by x and y and the inclination θ of the link CD with X -axis or link AB. In other words, we can say that each link of a planar mechanism has three degrees of freedom before it is connected to any other link. But when the link CD is connected to the link AB by a turning pair at A, as shown in Fig. 2.17 (b), the position of link CD is now determined by a single variable θ and thus has one degree of freedom.



(a) (b)

Fig. 2.17. Links in a plane motion

From above, we see that when a link is connected to a fixed link by a turning pair (*i.e.* lower pair) two degrees of freedom are destroyed (removed). This may be clearly understood from Fig. 2.18, in which the resulting four bar mechanism has one degree of freedom (*i.e.* F =1).



(a) $n = 9$ (b) $n = 7$ (c) $n = 5$ (d) $n = 3$ (e) $n = 1$

Fig. 2.18. Four bar mechanism

Based on above discussions, expression for degree of freedom of a planar kinematic chain, consisting of lower pairs (of d.o.f. =1) only, is given by-

$$F = 3n - 2l,$$

where n is a number of mobile links, l is a number of lower pairs.

In case of a mechanism which is obtained from a chain by fixing one link, number of mobile links reduces to $(n-1)$ and therefore, expression for degrees of freedom of a mechanism, consisting of lower pairs only, is given by-

$$F = 3(n-1) - 2l. \qquad (2.1)$$

Equation (2.1) is known as Grubler's equation, and is one of the most popular mobility equations.

Therefore, Fig. 2.18 illustrates the process of losing degrees of freedom, each time a turning pair is introduced, *i.e.* adding constraints, between two unconnected links.

Just as a lower pair (linear motion lower pair) cuts down 2 d.o.f., a higher pair cuts only 1 d.o.f. (this is because invariably rolling is associated with slipping, permitting 2 d.o.f.). Hence equation (2.1) can be further modified to include the effect of higher pairs also. Thus, for mechanism having lower and higher pairs

$$F = 3(n-1) - 2l - h, \qquad (2.2)$$

where h is a number of higher pairs.

Equation (2.2) is the modified Grubler's equation. It is also as Kutzbach criterion for the mobility of a planar mechanism. It would be more appropriate to define, in equations (2.1) and (2.2), l to be the number of pairs of 1 d.o.f. and h to be number of pairs of 2 d.o.f.

Spatial mechanisms do not incorporate any restriction on the relative motions of the particles. A spatial mechanism may have particles describing paths of double curvature. Grubler's criterion was originally developed for planar mechanisms. If similar criterion is to be developed for spatial mechanisms, we must remember that an unconnected link has six in place of 3 degrees of freedom. As such, by fixing one link of a chain the total d.o.f. of $(n-1)$ links separately will be $6(n-1)$. Again a revolute and prismatic pair would provide 5 constrains (permitting 1 d.o.f), rolling pairs will provide 4 constraints, and so on. Hence, taking into account the tab 2.1, an expression for d.o.f. of a closed spatial mechanism can be written as:

$$F = 6(n-1) - 5l_1 - 4l_2 - 3l_3 - 2l_4 - l_5, \qquad (2.3)$$

where N =total number of links,

$l_1 =$ number of pairs (joints) providing 5 constraints,

$l_2 =$ number of pairs providing 4 constraints,

$l_3 =$ number of pairs providing 3 constraints,

$l_4 =$ number of pairs providing 2 constraints, and

$l_5 =$ number of pairs providing only one constraint.

2.7. Application of Kutzbach Criterion to Plane Mechanisms

We have discussed that Kutzbach criterion for determining the number of degrees of freedom (F) of a plane mechanism is

$$F = 3(n-1) - 2l - h.$$

The number of degrees of freedom for some simple mechanisms having no higher pair (*i.e.* $h=0$), as shown in Fig. 2.19, are determined as follows:

Example 2.1. Find out degrees of freedom (F) of mechanisms shown in Fig. 2.17.

1. The mechanism, as shown in Fig. 2.19 (a), has three links and three lower

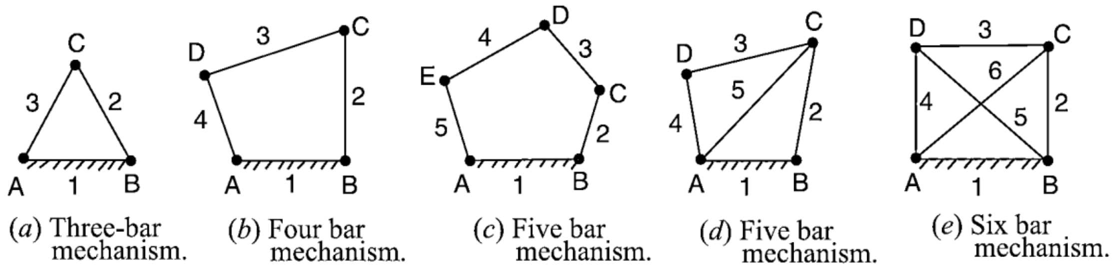


Fig.2.19. Plane mechanisms

pairs, *i.e.* $l=3$ and $n=3$,

$\therefore$ $$F=3(3-1)-2\times3=0.$$

2. The mechanism, as shown in 2.19 (b), has four links and four pairs, *i.e.* $l=4$ and $n=4$,

$\therefore$ $$F=3(4-1)-2\times4=1.$$

3. The mechanism, as shown in Fig. 2.19 (c), has five links and five pairs, *i.e.* $l=5$, and $n=5$,

$\therefore$ $$F=3(5-1)-2\times5=2.$$

4. The mechanism, as shown in Fig. 2.19 (d), has five links and six pairs (because there are two pairs at B and D, and four equivalent pairs at A and C), *i.e.* $l=5$ and $n=6$,

$\therefore$ $$F=3(5-1)-2\times6=0.$$

5. The mechanism, as shown in Fig. 2.19 (e), has six links and eight pairs (because there are two pairs separately at A, B, C and D), *i.e.* $l=6$ and $n=8$,

$\therefore$ $$F=3(6-1)-2\times8=-1.$$

Therefore, it may be noted that

(a) When $F=0$, then the mechanism forms a structure and no relative motion between the links is possible, as shown in Fig. 2.19 (a) and (d).

(b) When $F=1$, then the mechanism can be driven by a single input motion, as shown in Fig. 2.19 (b)

(c) When $F=2$, then two separate input motions are necessary for the mechanism, as shown in Fig. 2.19 (c).

(d) When $F=-1$ or less, then there are redundant constraints in the mechanism (chain) and it forms indeterminate structure, as shown in Fig. 2.19 (e).

Let's consider other examples.

Example 2.2. Find out degrees of freedom of mechanism shown in Figs. 2.20(a),(b),(c),(d) and (e).

Solution: (a) Here $n=9; l=11$,

$\therefore$ $$F=3(9-1)-2(11)=2.$$

(b) Here $n = 8, l = 9 + 2 = 11$,

$\therefore$ $$F = 3(8-1) - 2(11) = -1.$$

i.e. the mechanism at Fig. 2.20(b) is a statically indeterminate structure.

(c) As in case (b), here too there are double joints as *A* and *B*. Hence

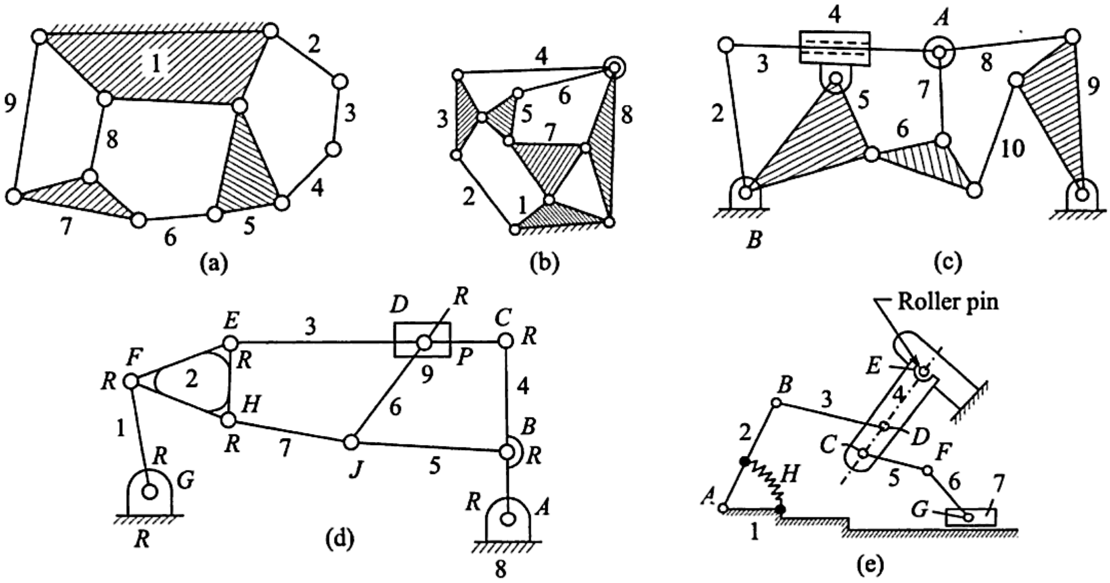


Fig. 2.20. Plane mechanisms

$$n = 10; l = 9 + 2(2) = 13,$$

$\therefore$ $$F = 3(10-1) - 2(13) = 1.$$

(d) The mechanism at Fig. 2.20(d) has three ternary links (links 2,3 and 4) and 5 binary links (links 1,5,6,7 and 8) and one slider. It has 9 simple turning pairs marked R, one sliding pair marked P and one double joint at J. Since the double joint J joints 3 links, it may be taken equivalent to two simple turning pairs. Thus, $n = 9; l = 11$,

$\therefore$ $$F = 3(9-1) - 2(11) = 2.$$

(e) The mechanism at Fig. 2.20(e) has a roller pin at E and a spring at H. The spring is only a device to apply force, and is not a link. Thus there are 7 links numbered through 7, one sliding pair, one rolling (higher) pairs at E besides 6 turning pairs

$\therefore$ $$n = 7; l = 7 \text{ and } h = 1,$$

$$F = 3(7-1) - 2(7) - (1) = 18 - 14 - 1 = 3.$$

Example 2.3. Find out degrees of freedom of the mechanism shown in Fig. 2.21 (a), (b).

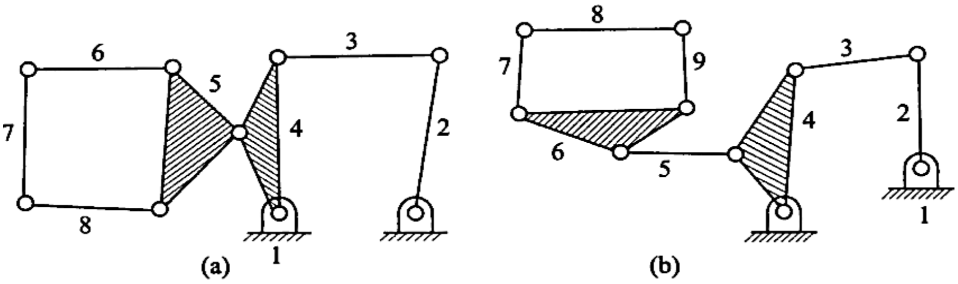


Fig. 2.21. Plane mechanisms

Solution: $(a) n = 8; l = 9$,

$\therefore$ $$F = 3(8-1) - 2(9) = 3.$$

(b) $n = 9, l = 10$,

$\therefore$ $$F = 3(9-1) - 2(10) = 4.$$

Example 2.4. Show that the automobile window glass guiding mechanism in Fig. 2.22 has a single degree of freedom

Solution: As numbered, there are total 7 links. There are seven revolute pairs between link pairs (1,2), (2,3), (3,4), (3,7), (4,6), (4,1) and (1,5). Besides, there is one sliding pair between links 6 and 7 and a geared pair between links 4 and 5.

Thus, $l = 8$ and $h = 1$,

$\therefore$ $F = 3(7-1) - 2(8) - 1 = 1.$

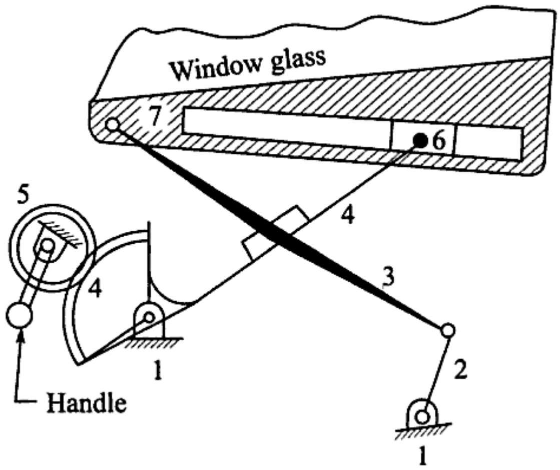


Fig. 2.22. Automobile window guidance linkage

2.8. Grubler's Criterion for Plane Mechanisms

The Grubler's criterion applies to mechanisms with only single degree of freedom pairs where the overall mobility of the mechanism is unity. Substituting in (2.2) $F = 1$ and $h = 0$, we have

$$1 = 3(n-1) - 2l \text{ or } 3n - 2l - 4 = 0.$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion. A little consideration will show that a plane mechanism with a mobility of 1 and only low pairs (of one degree of freedom) cannot have odd number of links. The simplest possible mechanism of this type are a four bar mechanism and a slider-crank mechanism in which $n = 4$ and $l = 4$.

Consider some cases when Grubler's equation gives incorrect results, particularly when

(1) the mechanism has a lower pair which could replaced by a higher pair, without influencing output motion;

(2) the mechanism has a kinematically redundant pair, and

(3) there is a link with redundant degree of freedom.

Inconsistency at (1) may be illustrated with the help of Figs. 2.23(a) and (b). Fig. 2.23(a) depicts a mechanism with three sliding pairs. According to Grubler's theory, this combination of links has a degree of freedom of zero. But by inspection, it is clear that the links have a constrained motion, because as the 2 is pushed to the left, link 3 is lifted due to wedge action. But the sliding pair between; links 2 and 3 can be replaced by a slip rolling pair (Fig. 2.23(b)), ensuring constrained motion. In the latter case, $n = 3, l = 2$ and $h = 1$ which, according to Grubler's equation, gives $F = 1$.

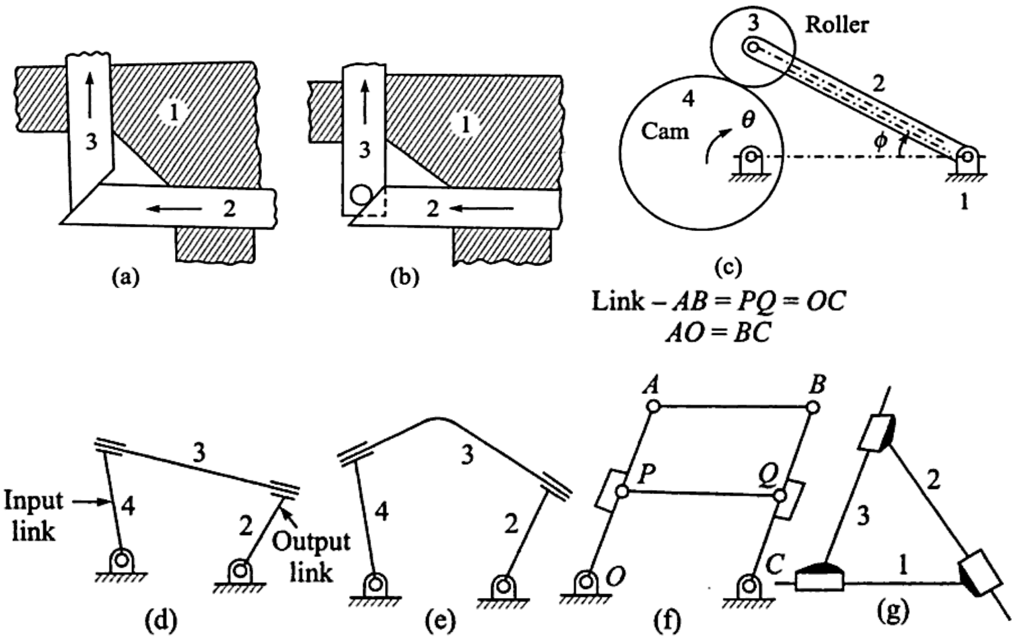


Fig. 2.23. Inconsistencies of Grubler's criterion

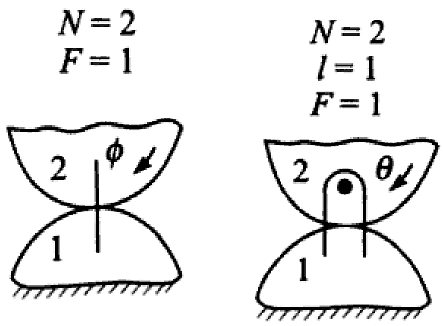
Fig. 2.23(c) demonstrates inconsistency at (2). The cam follower mechanism has 4 links, 3 turning pairs and a rolling pair, giving d.o.f. as 2. However, a close scrutiny reveals that as a function generator, oscillatory motion of follower is a unique function of cam rotation, i.e. $\phi = f(\theta)$. In other words, d.o.f. of the above mechanism is only 1. It may be noted, however, that the function of roller in this case is to minimize friction; it does not in any way influence the motion of follower. For instance, even if the turning pair between follower and roller is eliminated (rendering roller to be an integral part of follower), the motion of follower will not be affected. Thus the kinematic pair between links 2 and 3 is redundant. Therefore, with this pair eliminated, $n = 3, l = 2$ and $h = 1$, gives d.o.f. as one.

If a link can be moved without producing any movement in the remaining links of mechanism, the link is said to have *redundant degree of freedom*. Link 3 in mechanism of Fig. 2.23 (d), for instance, can slide and rotate without causing any movement in links 2 and 4. Since the Grubler's equation gives d.o.f. as 1, the loss due to redundant d.o.f. of link 3 implies effective d.o.f. as zero, and Fig. 2.23 (d) represents a locked system. However, if link 3 is bent, as shown in Fig. 2.23 (e), the link 3 ceases to have redundant d.o.f. and constrained motion results for the mechanism. Fig. 2.23 (f) shows a mechanism in which one of the two parallel links AB and PQ is redundant link, as none of them produces additional constraint. By removing any of the two links, motion remains the same. It is logical therefore to consider only one of the two links in calculating degrees of freedom. Another example where Grubler's equation gives zero mobility is the mechanism shown in Fig. 2.23 (g), which has a constrained motion.

2.9. Grubler's Criterion Application for Mechanisms with Higher Pairs

As against one degree freedom of relative motion permitted by turning and sliding pairs, higher pairs may permit a higher number of degrees of freedom. *Each such higher pair is equivalent to as many lower pairs as the number of degrees of freedom of relative motion permitted by the given higher pair*. This is elaborated for different types of higher pairs, as discussed below:

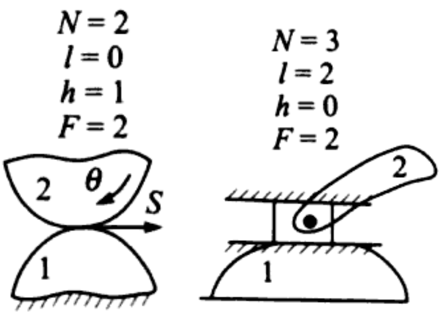
(a) *Rolling Contact without Sliding*. This allows only one d.o.f. of relative motion as only relative motion of rotation exists. A pare rolling type of joint can therefore be taken equivalent to lower pair with one d.o.f.(Fig. 2.24) The lower pair equivalent for instantaneous velocity is given by a simple hinge joint at the relative instant centre which is the point of contact between rolling links. Note that instantaneous velocity implies that in case a higher pair is replaced by a lower pair equivalent, the instantaneous relative velocity between the connecting links remains the same, but the relative acceleration may, in general, change.



(a) (b)

Fig 2.24. Rolling contact

(b) *Roll-Slide Contact*. Due to sliding motion associated with rolling only one out of three planar motions is constrained Fig. 2.25 (a). Thus, lower pair equivalence for instantaneous velocity is given by a slider and pin joint combination between the connected links Fig. 2.25 (b). This implies degrees of freedom of relative motion. Such a joint is also taken care of, in Grubler's equation, by making contribution to the term h.

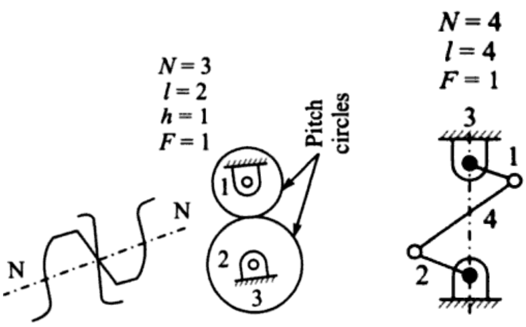


(a) (b)

Fig. 2.25 Roll-Slide contact

(c) *Gear-Tooth Contact (Roll-Slide)*. Gear tooth contact is a roll-slide pair and therefore makes a contribution to the term h in Grubler's equation. Thus, on account of two turning pairs at gear centers together with a higher pair at contacting teeth (Fig. 2.26 (a)),

$$F = 3(3-1) - 2(2) - 1 = 1.$$

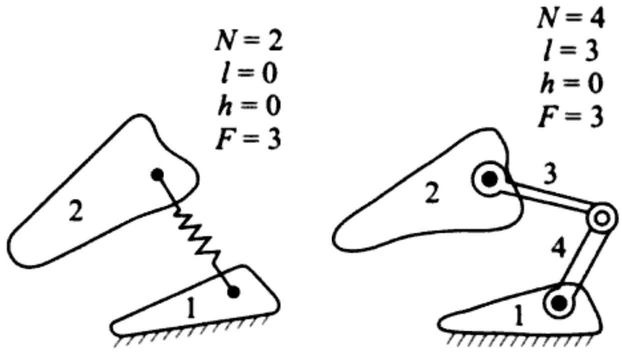


(a) (b)

Fig. 2.26. Gear-tooth contact

Lower pair equivalent for instantaneous velocity of such a pair is a 4-bar mechanism with fixed pivots at gear centers and moving pivots at the centers of curvature of contacting tooth profiles (Fig. 2.26 (b)). In case of involute teeth, these centers of curvature will coincide with points of tangency of common tangent drawn to base circles of the two gears. Such a 4-bar mechanism retains that d.o.f. equal to 1.

(d) *A Spring Connection*. Purpose of a spring is to exert force on the connected links, but it does not participate in relative motion between connected links actively. Since the spring permits elongation and contraction in length, a pair of binary links, with a turning pair connecting them, can be considered to constitute instantaneous velocity equivalent lower pair mechanism. A pair of binary links with a turning pair permits variation in distance between their other ends (unconnected), and allows same degree of freedom of relative motion between links connected by the spring (for $n = 4, l = 3, F = 3$). It may be noted that in the presence of spring, $(n = 2, \iota = 0, h = 0)$ the d.o.f. would be 3.

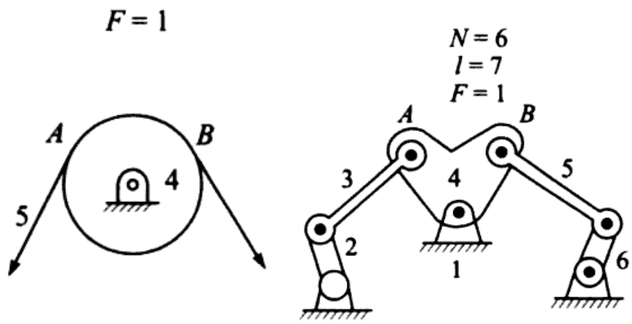


(a) (b)

Fig. 2.27. Spring Connection

(e) *The Belt and Pulley or Chain and Sprockets Connection.* When the belt or chain is maintained tight, it provides planar connections (Fig. 2.28 (a)). Instantaneous velocity, lower pair equivalent can be found in a ternary link with three pin joints (sliding is not allowed) as in (Fig. 2.28 (b)). It can be verified that d.o.f. of equivalent six bar linkage is

$$F = 3(6-1) - 2(7) = 1.$$



(a) (b)

Fig. 2.28. Belt and pulley connection

Example 2.5 Find out degrees of freedom of mechanisms shown in Fig. 2.29 (a),(b) and (c).

Solution: (a) In the case of undercarriage mechanism of aircraft in Fig.2.29 (a), we note that

Total number of pairs of single $F = 11$.

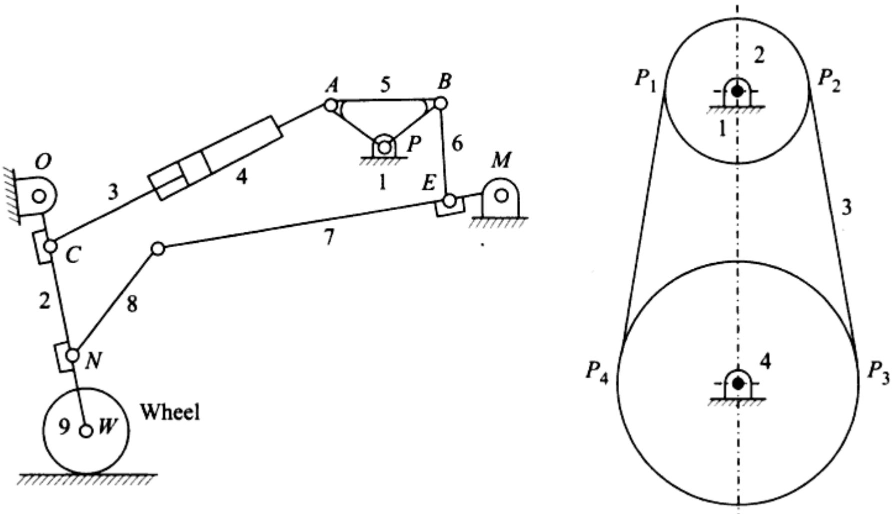
Higher pair of 2 d.o.f. (between wheel and runway) = 11.

$$F = 3(9-1) - 2(11) - 1(1) = 1.$$

(b) In the case of belt-pulley drive, assuming the belt to be tight, the four links are marked as 1, 2, 3 and 4. The two distinct lower (turning) pairs are pivots of pulley 2 and 4. The points P_1, P_2, P_3 and P_4 at which belt enters/leaves pulley, constitute 4 higher pairs. Thus

$$n = 4; l = 2; h = 4.$$

Therefore, $$F = 3(4-1) - 2(2) - 4 = 1.$$



(a) Undercarriage mechanism of aircraft (b) belt-pulley drive
Fig. 2.29. Degree of freedom of mechanisms

(c) In the case of mechanism at Fig. 2.29 (c), there is a double joint between links 6, 7, and 10. Therefore, this joint is equivalent to two simple joints. Besides above, there are 13 turning pairs.

Hence,

$$n = 12; l = 13 + 2 = 15.$$

Therefore,

$$F = 3(12-1) - 2(15) = 3.$$

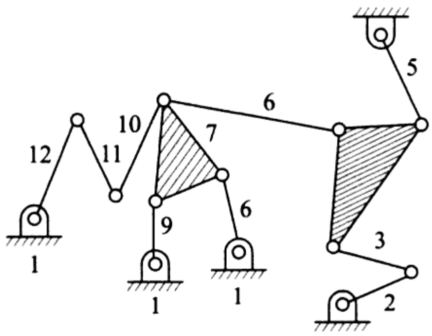


Fig. 2.29 (c). Mechanism with double pin joint

2.10. Equivalent Mechanisms

Equivalent linkages are commonly employed to duplicate instantaneously the position, velocity, and perhaps acceleration of a direct-contact (higher pair) mechanism by a mechanism with lower pairs (say, a four-bar mechanism). The dimensions of equivalent mechanisms are obviously different at various positions of given higher paired mechanism. This is evident because for every position of a higher paired mechanism, different equivalent linkages are expected.

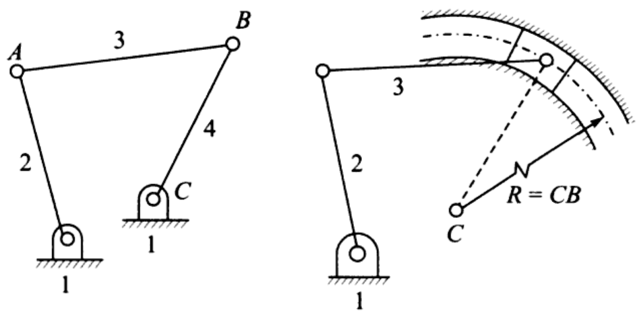
Much of the developments in kinematics in the subject of theory of machines are centered on four-bar mechanism. Some of the reasons are as under:

(1) A four-bar mechanism is the simplest possible lower paired mechanism and is widely used.

(2) Many mechanisms which do not have any resemblance with a four-bar mechanism have four bars for their basic skeletons, so a theory developed for the four-bar applies to them also.

(3) Many mechanisms have equivalence in four-bar mechanism in respect of certain motion aspects. Thus, as far as these motions are concerned, four-bar theory is applicable.

(4) Several complex mechanisms have four-bar loop as a basic element. Theory of four-bar mechanism is, therefore, useful in the design of these mechanisms.



(a) (b) (c)

Fig. 2.30. Equivalent mechanisms (kinematically identical mechanisms having the 4-bar basic skeleton)

Point (2) above, is illustrated in Figs. 2.30 (a), (b) and (c). In Fig. 2.30 (b), the link 4 in Fig. 2.30 (a) replaced by a curved slot and slider, with slot radius equal to link length. In Fig. 2.30 (c) the link 3 is replaced by a slider, sliding in curved slotted link 4 ensuring relative motion of rotation of pinned and *A* relative to *B*.

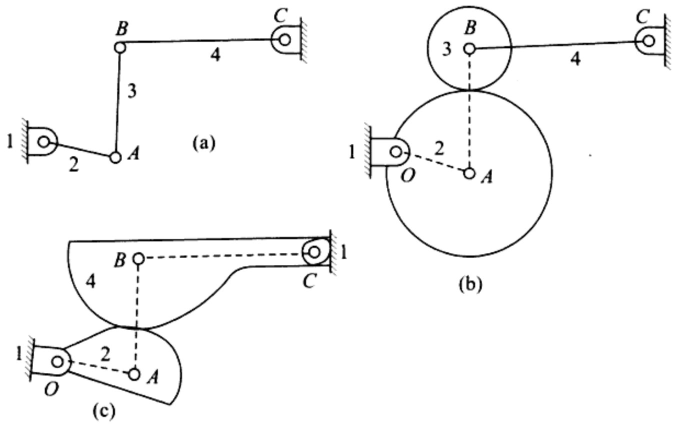
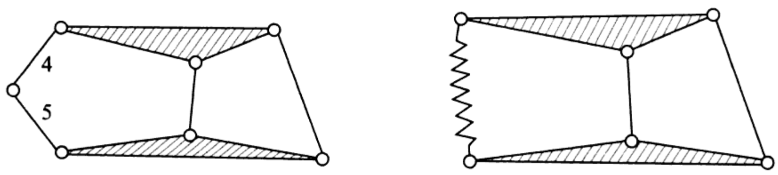


Fig. 2.31. Mechanisms having identical relative motions between links 2 and 4

Point (3) is illustrated in Figs. 2.30 (a), (b) and (c). Mechanisms in which relative motion between driver and driven links 2 and 4 is identical are illustrated in Fig. 2. 31.

In Fig. 2.31 (b) the centers of curvature of circular cam and roller constitute the end point of link *AB*; link 3 becomes roller and link 2 becomes circular cam. For $\text{d.o.f.} = 1$, however, the rolling pair in (b) should be without slip.

Extension and compression in a spring is comparable to variation in length between the turning pairs accomplished by a pair of binary links connected through another turning pair. For instance pair of binary links 4 and 5 of a Stephenson's chain can be replaced by a spring to obtain an equivalent mechanism. This is shown in Figs. 2.32 (a) and (b).



(a) (b)

Fig. 2.32. Spring to replace a pair of binary links and ternary pairs

When the belt or chain is maintained tight, a ternary link with three turning pairs is the instantaneous-velocity equivalent lower pair connection to the belt and pulley (sliding/slipping is disallowed).

2.11. Inversion of Mechanism

We have already discussed that when one of links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain is known as *inversion of the mechanism*. It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

The part of a mechanism which initially moves with respect to the frame or fixed link is called *driver* and that part of the mechanism to which motion is transmitted is called *follower*. Most of the mechanisms are reversible, so that same link can play the role of a driver and follower at different times. For example, in a reciprocating steam engine, the piston is the driver and flywheel is a follower while in a reciprocating air compressor, the flywheel is a driver.

Important aspects of the concept of inversion can be summarized as under:

1. The concept of inversion enables us to categorize a group of mechanisms arising out of inversions of a parent kinematic chain as a family of mechanisms. Members of this family have a common characteristic in respect of relative motion.
2. In case of direct inversions, as relative velocity and relative acceleration between two links remain the same, it follows that complex problems of velocity/acceleration analysis may often be simplified, by considering a kinematically simpler direct inversion of the original mechanism.
3. In many cases of inversions by changing proportions of lengths of links, desirable features of the inversion may be accentuated and many useful mechanisms may be developed.

The most important kinematic chains are those which consist of four lower pairs each pair being a sliding or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:

1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.

These kinematic chains are discussed, in detail, in the following articles.

2.12. Four Bar Chain or Quadric Cycle Chain

We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained. The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. 2.33. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths. According to *Grashof's law* for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links. Thus, if s and l be the lengths of shortest and longest links respectively and p and q be the remaining two link-lengths, then one of the links, in particular the shortest link, will rotate continuously relative to the other three links, if and only if

$$s + l \leq p + q .$$

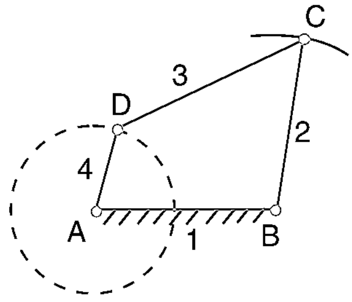


Fig. 2.33. Four bar mechanism

If this inequality is not satisfied, the chain is called non-Grashof chain in which none of the links can have complete revolution relative to other links. It is important to note that the Grashof's law does not specify the order in which the links are to be connected. Thus any of the links having length l, p and q can be the link opposite to the link of length s. A chain satisfying Grashof's law generates three distinct inversions only. A non-Crashof chain, on the other hand, generates only one distinct inversion, namely the "Rocker-Rocker mechanism".

A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof 's law. Such a link is known as *crank* or *driver*. In Fig. 2.33 AD (link 4) is a crank. The link BC (link 2) which makes a partial rotation or oscillates is known as *lever* or *rocker* or *follower* and the link CD (link 3) which connects the crank and lever is called *connecting rod* or *coupler*. The fixed link AB (link 1) is known as *frame* of the mechanism. When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view:

1. Double crank mechanism (*Coupling rod of a locomotive*). The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links is shown in Fig. 3.34.

In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant center to center distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

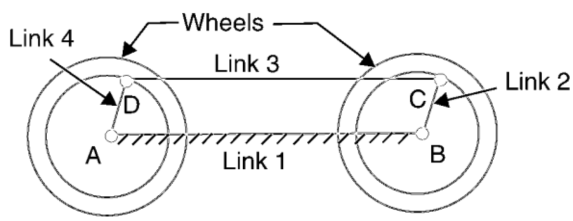


Fig. 2.34. Coupling rod of a locomotive

*2. Crank-rocker mechanism (Beam engine).*A part of the mechanism of a beam engine (also known as cranks and lever mechanism), which consists of four links, is shown in Fig. 3.35. In this mechanism, when the crank rotates about the fixed centre *O*, the lever oscillates about a fixed centre *C*. The end *D* of the lever *BCD* is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

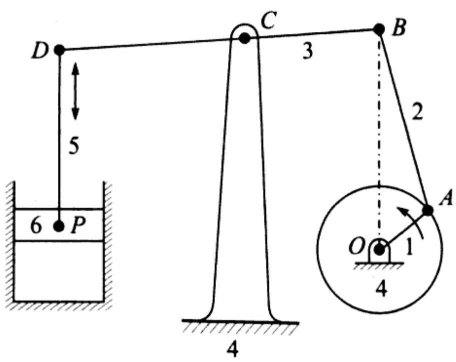


Fig. 2.35. Beam engine mechanism

3. *Double rocker mechanism*. When the link, opposite to the shortest link is fixed, a double rocker mechanism results. None of the two links (driver and driven) connected to the frame can have complete revolution but the coupler link can have full revolution (Fig. 2.36)).

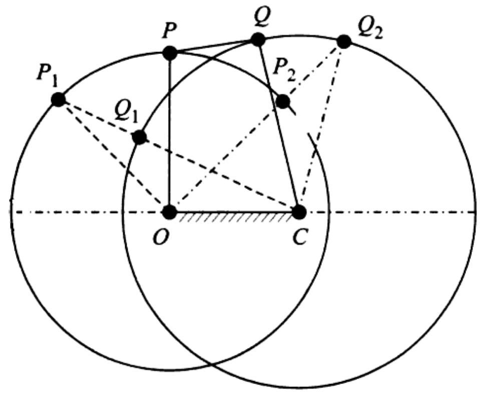


Fig. 2.36. Double rocker mechanism

Example 2.6. Figure 2.37 shows a planar mechanism with link-lengths given in some unit. If slider A is the driver, will link CG revolve or oscillate? Justify your answer.

Solution: The loop formed by three links DE, EF and FD represents a structure. Thus the loop can be taken to represent a ternary link.

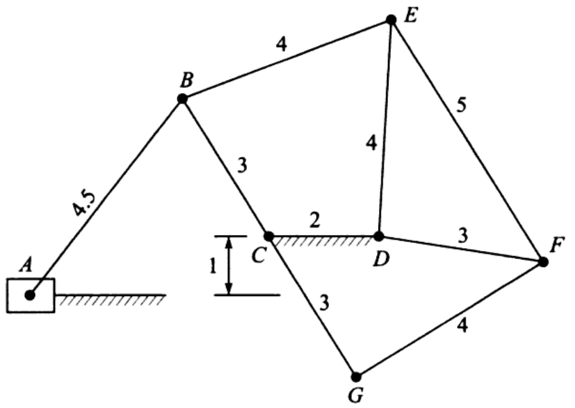


Fig. 2.37. Application of Grashow's law

In the 4-link loop $CDEB$, $s=2; l=4$; and $p+q=7$.Thus the 4-link loop portion $CDEB$ satisfies Grashof's criterion. And as the shortest link CD is fixed, link CB is capable of complete revolution. Also, 4-link loop $GDFG$ satisfies Grashof's criterion $(l+s=p+q)$ and the shortest link CD is fixed. Thus whether considered a part of 4-link loop $CDFB$ or that of $CDFG$, link BCG is capable of full revolution

Example 2.7. In a 4-bar mechanism, the lengths of driver crank, coupler and follower link are 150 mm, 250 mm and 300 mm respectively. The fixed link-length is L_0. Find the range of values for L_0, so as to make it a –

(1) Crank-rocker mechanism, (2) Crank-crank mechanism.

Solution:(1) For crank-rocker mechanism the conditions to be satisfied are:

(a) Link adjacent to fixed link must be the smallest link and, (b) $s+l \le p+q$.

We have to consider both the possibilities, namely, when L_0 is the longest link and when L_0 is not the longest link.

When L_0 is the longest link, it follows from Grashof's criterion,

$$L_0 + 150 \le 250 + 300 \text{ or } L_o \le 400mm$$

When L_0 is not the longest link, it follows from Grashof's criterion,

$$300 + 150 \le L_0 + 250 \text{ or } L_0 \ge 200$$

Thus, for crank-rocker mechanism, range of values for L_0 is

$$200 \le L_0 \le 400mm$$

(2) For crank-crank mechanism, the conditions to be satisfied are

(a) Shortest link must be the frame link and, (b) $s + l \le p + q$.

Thus, $$L_0 + 300 \le 150 + 250$$

$$\text{or } L_0 \le 100mm$$

2.13. Inversion of Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa.

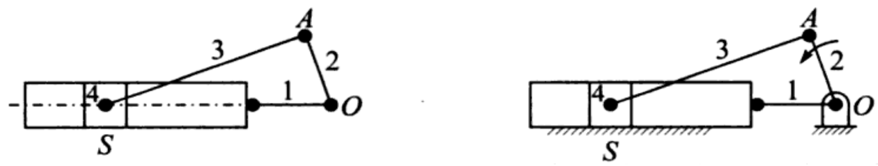
We know that by fixing, in turn, different links in a kinematic chain, an inversion is obtained and we can obtain as many mechanisms as the links in a kinematic chain. It is thus obvious, that four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

A slider crank chain is as shown in Fig. 2.38(a).

First Inversion. It is obtained by fixing link 1 of the chain and the result is the crank-slider mechanism as shown in Fig. 2.38(b). This mechanism is very commonly used in I.C. engines, steam engines and reciprocating compressor mechanism.

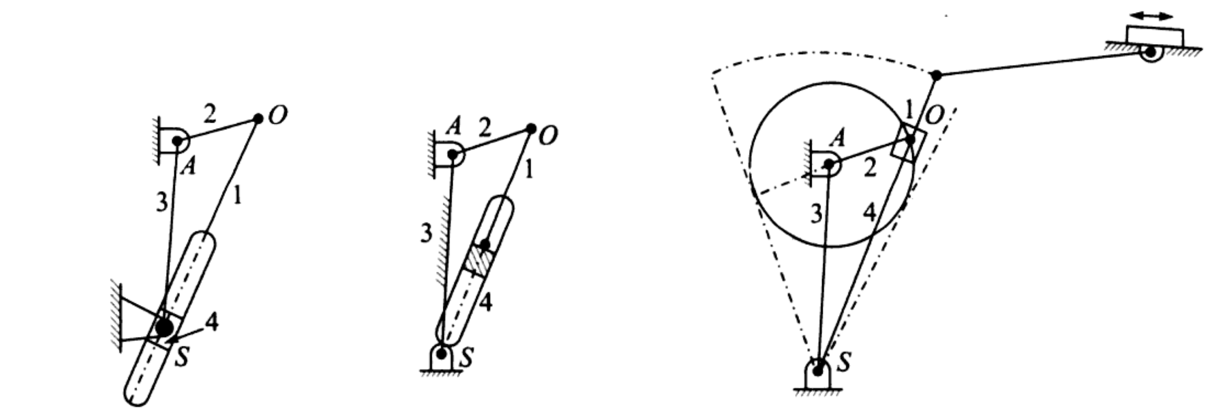
Second Inversion. It is obtained by fixing link 3, the connecting rod. The mechanism obtained by 'verbatim inversion', as shown in Fig. 2.39(a), has some practical difficulties. For instance, the oscillating cylinder will have to be slotted for clearing the pin through which slider is pivoted to frame. The problem may be resolved if one remembers that any suitable alteration in shapes of links ensuring same type of pairs between links 3 and 4 and also between links 1 and 4, is permissible. This gives an inversion at Fig. 2.39 (b). The resulting mechanism is oscillating cylinder engine mechanism. It is used in hoisting engine mechanism and also in toys. In hoisting purposes its chief advantage lies in its compactness of construction as it permits simple scheme of supplying steam to the cylinder.

Second application of the above inversion lies in 'Slotted Lever Quick Return Mechanism', shown in Fig. 2.39 (c). The extreme position of lever 4 is decided by the tangents drawn from lever-pivot to the crank-circle on either side. Corresponding positions of crank 1 include angels and, which correspond to cutting stroke angle and return stroke angle.



(a) Parent slider-crank chain (b) First inversion: slider-crank mechanism

Fig. 2.38. First inversion of a slider crank mechanism



(a) Verbatim inversion (b)Actual inversion oscillating cylinder engine mechanism (c) Actual inversion slotted lever quick return mechanism

Fig. 2.39. Second inversion of a slider crank mechanism

Third Inversion. The third inversion is obtained by fixing crank 2. It is the slider-crank equivalent of *Drag-link mechanism* and forms the basis of *Whitworth Quick Return Mechanism.* Basic inversion is given by portion OAS. To derive advantage however, the slotted link 1 is extended up to P and here it is connected to reciprocating tool-post through a connecting link PQ and two turning pairs. The cutting stroke angle θ_C and return stroke angle θ_R are shown in Fig. 2.40.

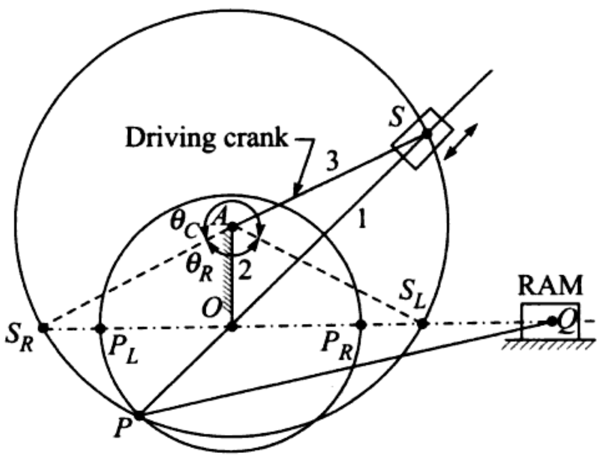


Fig. 2.40. Third inversion of a slider crank mechanis
Whitworth quick return mechanism

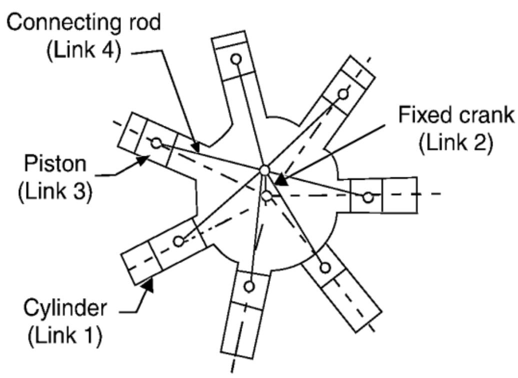
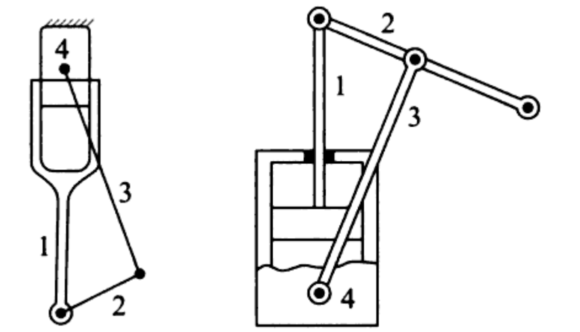


Fig. 2.41. Rotary internal combustion engine

A yet another application of third inversion is in Rotary internal combustion engine or Gnome engine (Fig. 2.41).

Fourth Inversion. The fourth inversion is obtained by fixing slider, the link 4. Fixing of slider implies that the slider should be position-fixed and also fixed in respect of rotation. The verbatim inversion is shown in Fig. 2.42(a). This form has certain practical difficulties. As explained earlier, the cylinder will have to be slotted so as to clear piston pin of connecting rod as cylinder slides past piston. To overcome this difficulty, the shapes of piston and cylinder are exchanged as shown in Fig.2.42 (b). This gives a hand pump mechanism. Lever 2 is extended.



(a) Verbatim inversion (b) Modified version - hand pump mechanism

Fig. 2.42. Forth inversion of a slider crank mechanism

2.14. Applications of Single Slider Crank Chain Inversion

Consider crank and slotted lever quick return motion mechanism in detail. This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link AC (*i.e.* link 3) forming the turning pair is fixed, as shown in Fig. 2.43. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed center C. A sliding block attached to the crankpin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram (*i.e.* R_1R_2) is perpendicular to AC produced.

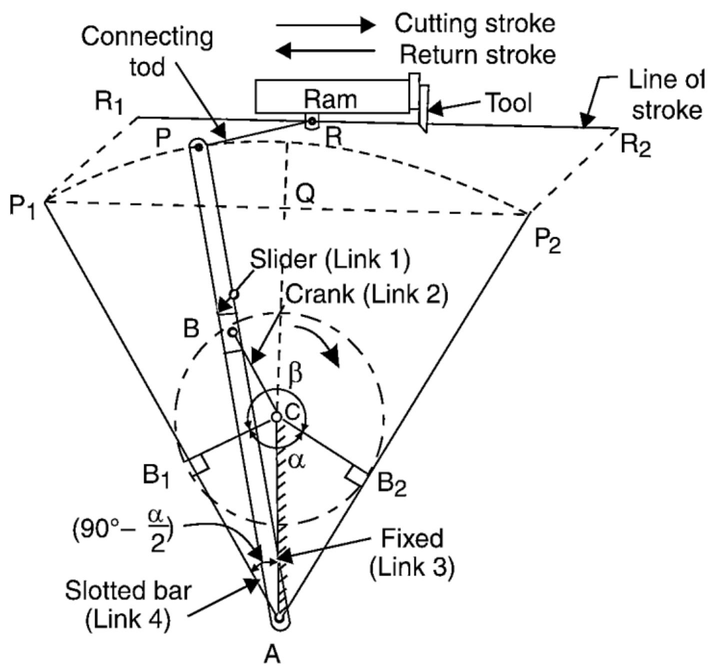


Fig. 2.43. Crank and slotted lever quick return motion mechanism

In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, we have

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^0 - \beta} = \frac{360^0 - \alpha}{\alpha}.$$

Since the tool travels a distance of R_1R_2 during cutting and return stroke, therefore travel of the tool or length of stroke is

$$R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin\angle P_1AQ = 2AP_1 \sin\left(90^0 - \frac{\alpha}{2}\right) = 2AP\cos\frac{\alpha}{2} =$$

$$= 2AP \times \frac{CB_1}{AC} = 2AP \times \frac{CB}{AC}.$$

From Fig. 2.43, we see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

Now let's analyze *Whitworth quick return motion mechanism.* In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in Fig. 2.44. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, *i.e.* along a line passing through D and perpendicular to CD.

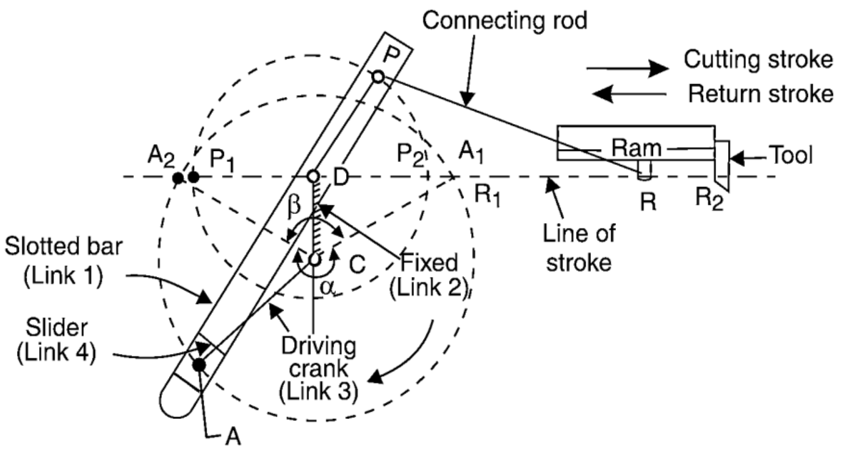


Fig. 2.44. Whitworth quick return motion mechanism.

When the driving crank CA moves from the position CA_1 to CA_2 (or the link DP from the position DP_1 to DP_2) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance $2PD$. Now when the driving crank moves from the position CA_2 to CA_1 (or the link DP from DP_2 to DP_1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end. A little consideration will show that the time taken during the left to right movement of the ram (*i.e.* during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from CA_2 to CA_1. Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}}=\frac{\alpha}{\beta}=\frac{\alpha}{360^0-\alpha}=\frac{360^0-\beta}{\beta}.$$

In order to find the length of effective stroke R_1R_2, mark $P_1R_1=P_2R_2=PR$. The length of effective stroke is also equal to $2PD$.

Example 2.8. A crank and slotted lever mechanism used in a shaper has a center distance of 300 mm between the center of oscillation of the slotted lever and the center of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.

Solution. Given: $AC=300$ mm; $CD_1=120$ mm. The extreme positions of the crank are shown in Fig. 2.45.

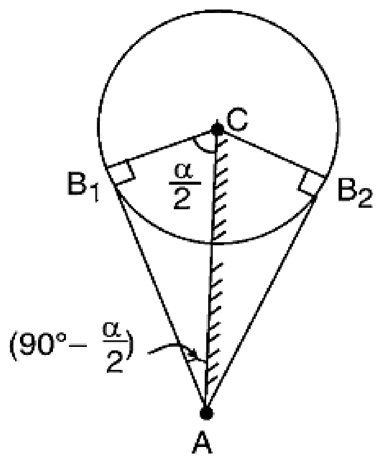


Fig. 2.45. Extreme positions of the crank

We know that

$$\sin\angle CAB_1=\sin\left(90^0-\alpha/2\right)=\frac{CB_1}{AC}=\frac{120}{300}=0.4,$$

whence $\angle CAB_1=90^0-\alpha/2=\sin^{-1}0.4=23.6^0$ or

$\alpha/2=90^0-23.6^0=66.4^0$ and $\alpha=2\times66.4^0=132.8^0$.

Finally we have

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}}=\frac{360^0-\alpha}{\alpha}=$$

$$=\frac{360^0-132.8^0}{132.8^0}=1.72.$$

Example 2.9. In a crank and slotted lever quick return motion mechanism, the distance between the fixed centers is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

Solution. Given: $AC=240$ mm; $CB_1=120$ mm; $AP_1=450$ mm

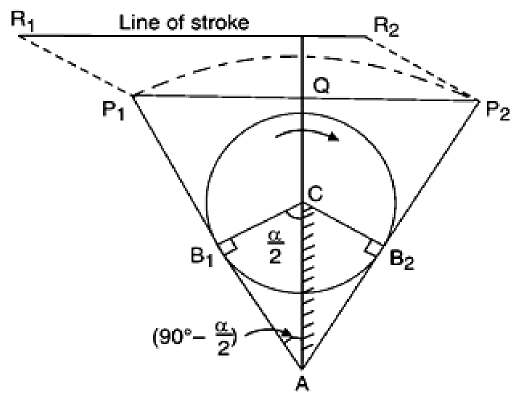


Fig. 2.46. Extreme positions of the crank

Let $\angle CAB_1$ be an inclination of the slotted bar with the vertical. The extreme positions of the crank are shown in Fig. 2.46. We know that

$$\sin\angle CAB_1 = \sin\left(90^0 - \frac{\alpha}{2}\right) = \frac{B_1C}{AC} = \frac{120}{240} = 0.5,$$

hence, $\angle CAB_1 = 90^0 - \dfrac{\alpha}{2} = \sin^{-1}0.5 = 30^0$.

We know that $90^0 - \alpha/2 = 30^0$, then $\alpha/2 = 90^0 - 30^0 = 60^0$ or $\alpha = 2\times 60^0 = 120^0$.

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360^0 - \alpha}{\alpha} = \frac{360^0 - 120^0}{120^0} = 2.$$

We know that length of the stroke,

$$R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1\sin\left(90^0 - \alpha/2\right) =$$
$$= 2\times 250\sin\left(90^0 - 60^0\right) = 900\times 0{,}5 = 450 \text{ mm}.$$

Example 2.10. Fig. 2.47 shows the layout of a quick return mechanism of the oscillating link type, for a special purpose machine. The driving crank BC is 30 mm long and time ratio of the working stroke to the return stroke is to be 1.7. If the length of the working stroke of R is 120 mm, determine the dimensions of AC and AP.

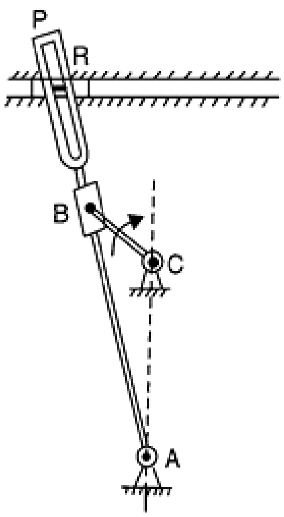


Fig. 2.47. Quick return mechanism

Solution. Given: $BC = 30$ mm; $R_1R_2 = 120$ mm; Time ratio of working stroke to the return stroke=1.7.

We know that

$$\frac{\text{Time of working stroke}}{\text{Time of return stroke}} = \frac{360 - \alpha}{\alpha} \text{ or}$$

$$1.7 = \frac{360 - \alpha}{\alpha}$$

Hence, $\alpha = 133.3^0$ or $\alpha/2 = 66.65^0$.

The extreme positions of the crank are shown in Fig. 2.48. From right angled triangle AB_1C, we find that

$$\sin\left(90^0 - \alpha/2\right) = \frac{B_1C}{AC} \text{ or}$$

$$AC = \frac{B_1C}{\sin\left(90^0 - \alpha/2\right)} = \frac{BC}{\cos\alpha/2}.$$

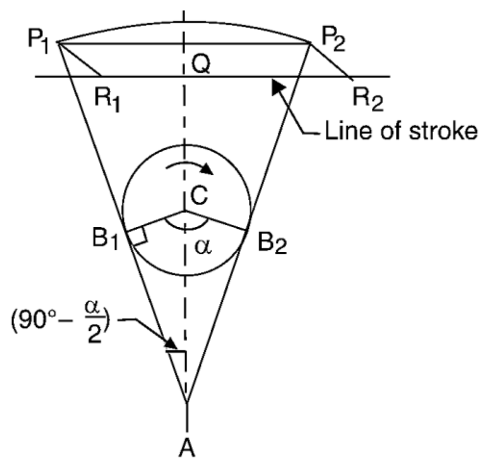


Fig. 2.48. Extreme positions of the crank

Since $B_1C = BC$ we obtain

$$AC = \frac{30}{\cos 66.65^0} = \frac{30}{0.3963} = 75.7 \text{ mm.}$$

We know that length of stroke,

$R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin\left(90^0 - \alpha/2\right) = 2AP_1 \cos\alpha/2$, but $AP_1 = AP$. Then

$$120 = 2AP\cos 66.65^0 = 0.7926AP \text{ and } AP = 120/0.7926 = 151.4 \text{ mm}$$

Example 2.11. In a Whitworth quick return motion mechanism, as shown in Fig. 2.49, the distance between the fixed centers is 50 mm and the length of the driving crank is 75 mm. The length of the slotted lever is 150 mm and the length of the connecting rod is 135 mm. Find the ratio of the time of cutting stroke to the time of return stroke and also the effective stroke.

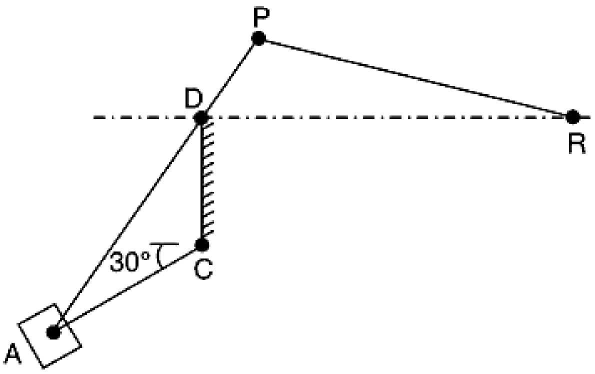


Fig. 2.49. Whitworth quick return motion mechanism

Solution. Given: $CD = 50$ mm; $CA = 75$ mm; $PA = 150$ mm ; $PR = 135$ mm

The extreme positions of the driving crank are shown in Fig. 2.50. From the geometry of the figure,

$$\cos\beta/2 = \frac{CD}{CA_2} = \frac{50}{75} = 0.667, \quad \text{then}$$

$\beta = 96.4^0$

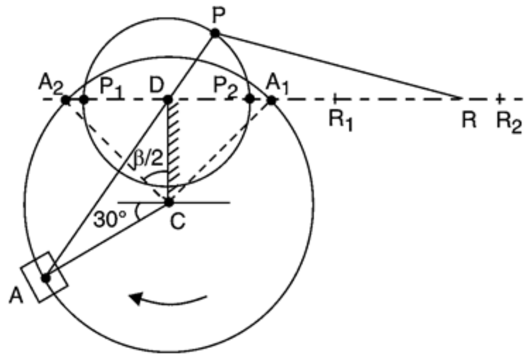


Fig. 2.50. Extreme positions of the driving crank

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360 - \beta}{\beta} = \frac{360^0 - 96.4^0}{96.4^0} = 2.735.$$

In order to find the length of effective stroke (*i.e.* R_1R_2), draw the space diagram of the mechanism to some suitable scale, as shown in Fig. 2.50. Mark $P_1R_2 = P_2R_2 = PR$. Therefore by measurement we find that,

Length of effective stroke is $R_1R_2 = 87.5$ mm

2.15. Inversions of Double Slider Crank Chain

A kinematic chain which consists of two turning pairs and two sliding pairs is known as *double slider crank chain,* as shown in Fig. 2.51. We see that the link 2 and link 1 form one turning pair and link 2 and link 3 form the second turning pair. The link 3 and link 4 form one sliding pair and link 1 and link 4 form the second sliding pair.

The following three inversions of a double slider crank chain are important from the subject point of view:

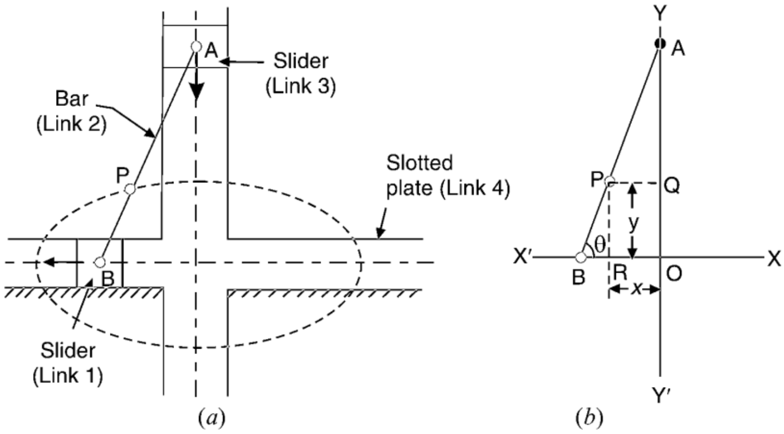


Fig. 2.51. Elliptical trammels

First inversion (Elliptical trammels). It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Fig. 2.51. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3 are known as sliders and form sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pair with links 1 and 3.When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface of link 4, as shown in Fig. 2.51 (*a*). A little consideration will show that AP and BP are the semi-major axis and semi-minor axis of the ellipse respectively. This can be proved as follows:

Let us take OX and OY as horizontal and vertical axes and let the link BA is inclined at an angle θ with the horizontal, as shown in Fig. 2.51 (*b*). Now the co-ordinates of the point P on the link BA will be

$$x = PQ = AP\cos\theta;\text{ and } y = PR = BP\sin\theta \text{ or}$$

$$\frac{x}{AP} = \cos\theta;\text{ and } \frac{y}{BP} = \sin\theta.$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1.$$

This is the equation of an ellipse. Hence the path traced by point P is an ellipse whose semi major axis is AP and semi-minor axis is BP.

If P is the mid-point of link BA, then $AP = BP$. The above equation can be written as

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(\mathrm{AP})^2} = 1 \text{ or}$$

$$x^2 + y^2 = (AP)^2.$$

This is the equation of a circle whose radius is AP. Hence if P is the mid-point of link BA, it will trace circle.

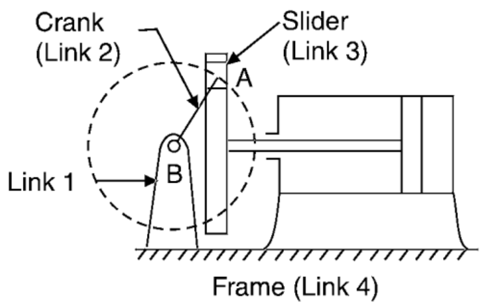


Fig. 2.52. Scotch yoke mechanism

Second inversion (Scotch yoke mechanism). This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig. 5.35, link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as center, the link4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.

Third inversion (Oldham's coupling). An Oldham's coupling is used for

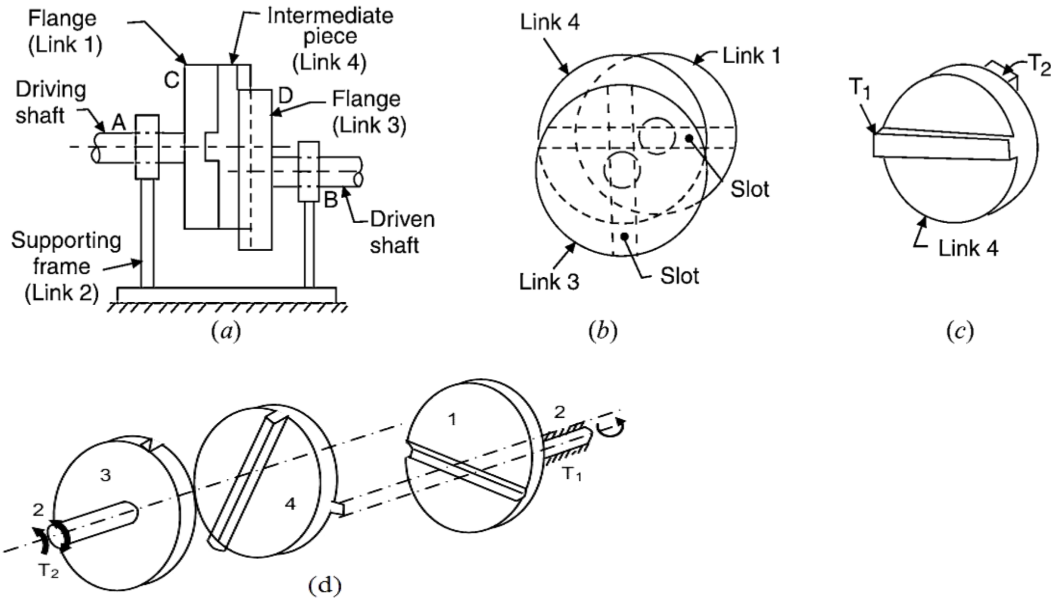


Fig. 2.53. Oldham's coupling

connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Fig. 2.53 (*a*). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces, as shown in Fig. 2.53 (*b*). The intermediate piece (link 4) which is a circular disc, have two tongues (*i.e.* diametrical projections) $T1$ and $T2$ on each face at right angles to each other, as shown in Fig. 2.53 (*c*). The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.

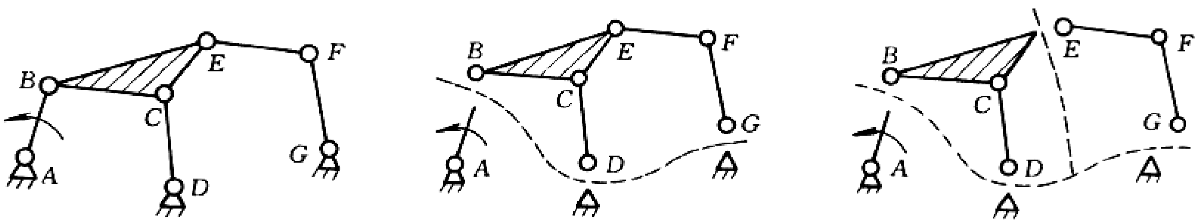
When the driving shaft A is rotated, the flange C (link 1) causes the intermediate piece (link4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange D (link 3) at the same angle and thus the shaft B rotates. Hence links 1, 3 and 4 have the same angular velocity at every instant. A little consideration will show that there is a sliding motion between the link 4 and each of the other links 1 and 3. If the distance between the axes of the shafts is constant, the center of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the center of the disc along its circular path.

Let ω be an angular velocity of each shaft in rad/s, and *r is a* distance between the axes of the shafts in meters. Then maximum sliding speed of each tongue (in m/s),

$$v = \omega \cdot r\,.$$

2.16. Assur-Artobolevsky Composition Principle and Structural Analysis

2.16.1. Composition Principle of Mechanisms



(a) (b) (c)

Fig. 2.54. Structural analysis of plane mechanism

The links and the kinematic pairs of a mechanism can be divided into two parts. The first part consists of the frame, the driver and the kinematic pair connecting the frame and the driver. Other links and pairs belong to the second part. The first part we will call the *basic mechanism* and the second part the system of driven links. The mechanism in Fig. 2.54 can for example be divided into two such parts as shown in Fig. 2.54 (b). During such division and classification, the sum of links, the sum and types of kinematic pairs do not change. The sum of the d.o.f. of the two parts should therefore be equal to the d.o.f. of the original mechanism.

We have learned that in any mechanism which has a determined motion, the number of drivers must be equal to the d.o.f. of the mechanism. In the basic mechanism, the driver is always connected to the frame by a lower pair. Every driver (and its corresponding lower pair) has one d.o.f. Thus the d.o.f. of the basic mechanism is equal to the number of drivers, or equal to the d.o.f. of the original mechanism. The d.o.f. of the system of driven links must thus be zero. In some cases, the system of driven links can be divided into smaller groups. If the d.o.f. of each group is zero and no group can be divided further into two or more zero-d.o.f. groups, then such groups are called Assur-Artobolevsky groups. For example, the system of driven links in Fig. 2.54 (b) can be further divided into two Assur groups as shown in Fig. 2.54 (c).

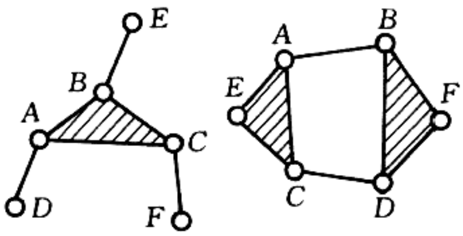
In each Assur group, one or more pairs are used to connect the links within the group. Such a pair is called an inner pair. For example, the pair *C* in the group *DCB* and the pair *F* in the group *GFE* are the inner pairs for the groups concerned. Some pairs in an Assur group are used to connect the group to kinematically determined links. Such pairs are called outer pairs. For example, the group *DCB* is connected to the kinematically determined links (the frame and the driver) by lower pairs *B* and *D*. The pairs *B* and *D* are therefore the outer pairs of the group *DCB*. When the group *DCB* is connected to the determined links by the outer pairs *D* and *B* a four-bar mechanism *ABCD* is created and all links in the group *DCB* become kinematically determined. The group *GFE* is then connected to the determined link *BCE* and the frame by lower pairs *E* and *G*. The pairs *E* and *G* are therefore the outer pairs of the group GFE. Note: the revolute E is not an outer pair of the group *DCB*. From the assembly order of the Assur groups, we can see that the group *DCB* is the first group, while the group *GFE* is the second group.

Hence, as mentioned above, we can see that any mechanism which has a determined motion can be assembled from a basic mechanism by connecting Assur groups to the determined links using outer pairs, group by group. This is the composition principle of mechanism. Only after the former Assur group is assembled can the later one be assembled.

2.16.2. Classification of Assur Group and Mechanism

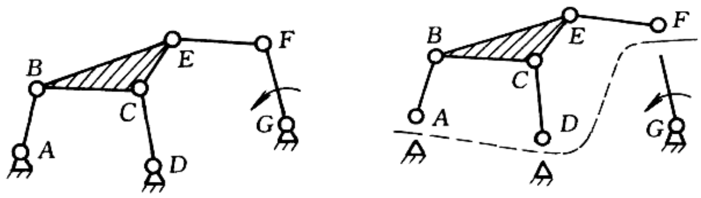
In a lower-pair Assur group, $F = 3n - 2l = 0$. Therefore, $l = \frac{3n}{2}$. Since l and n are integers, the number n of links must be even. The groups in Fig. 2.54 (c) are the simplest lower-pair Assur groups in which there are two links and three pairs. If n = 4, the lower-pair Assur group has two different constructions as shown in Fig. 2.55. In Fig. 2.55 (a), lower pairs *A*, *B* and *C* are used to connect links within the group. They are the inner pairs of the group. The group will be connected to determined links by lower pairs *D*, *E* and *F*. Thus, the lower pairs *D*, *E* and *F* are the outer pairs of the group. In Fig. 2.55 (b), lower pairs *A*, *B*, *C* and *D* are the inner pairs, while the lower pairs *E* and *F* are the outer pairs. Assur groups have different grades according to different number of links and different structure. The groups in Fig. 2.54 (c), Fig. 2.55 (a) and Fig. 2.55 (b) are classified as grade II, III and IV Assur groups, respectively.

For the same kinematic chain, the composition can be changed if the frame



(a) (b)

Fig. 2.55.



(a) (b)

Fig. 2.56.

and/or the driving link is changed. For example, the kinematic chain in Fig. 2.56 (a) is the same as that in Fig. 2.54 (a) but the driver in Fig. 2.56 (a) is the link GF. The mechanism in Fig. 2.56 (a) is then composed of a basic mechanism and a grade III Assur group, as shown in Fig. 2.56 (b).

The grade of a mechanism is defined as the highest grade of the Assur group in the mechanism. Hence the mechanism in Fig. 2.54 is a grade II mechanism, while the mechanism in Fig. 2.56 is of grade III. The basic mechanism is sometimes called the grade I mechanism, e.g., a ceiling fan (consisting of only a single rotating link) is a grade I mechanism.

If all pairs in Assur group are revolute pairs, the group is called the basic form of Assur group. If one or more revolute pairs are replaced by sliding pairs, some derivative forms of Assur groups will be created. The group name, schematic diagram, inner pair and outer pairs of some commonly used grade II Assur groups are shown in Fig. 2.57. The links in dashed lines are the kinematically determined ones.

2.16.3. Structural Analysis

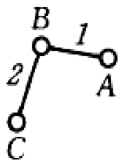
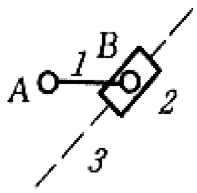
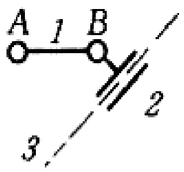
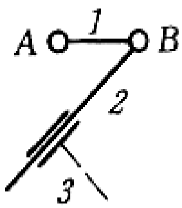
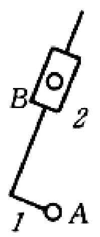
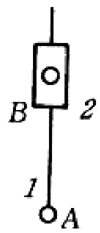
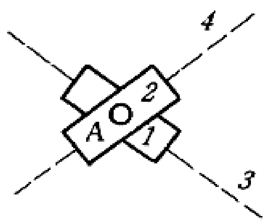
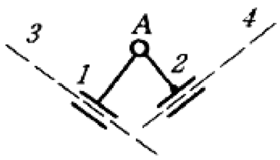
As mentioned above, a mechanism is assembled starting with the basic mechanism and adding Assur groups to the determined links using the outer pairs, group by group. The purpose of structural analysis is to disconnect the Assur groups from the mechanism and to determine their types and assembly order. The steps of structural analysis for grade II linkage mechanisms are as follows.

(1) Delete all redundant constraints.

(2) The frame and the basic mechanism are determined links. Other links are undetermined links.

(3) From all undetermined links that are connected to determined links, choose two connected links. These two links constitute a grade II Assur group.

Table 2.2. Commonly used grade II Assur groups

Group name	*RRR*	*RRP*	*RPR*	*PRP*
Schematic diagram		  	  	 
Inner pair	*B*	revolute pair *B*	sliding pair	revolute pair A

Outer pairs	*C, A*	*A*, sliding pair	*A*, revolute *B*	sliding pair 1-3 sliding pair 2-4

The pair connecting these two links is the inner pair of the group. The two pairs by which the group is connected to the determined links are the two outer pairs of the group.

(4) When the group is connected to the determined links by the outer pairs, all links in the group become kinematically determined. Now repeat step (3) until all links become kinematically determined.

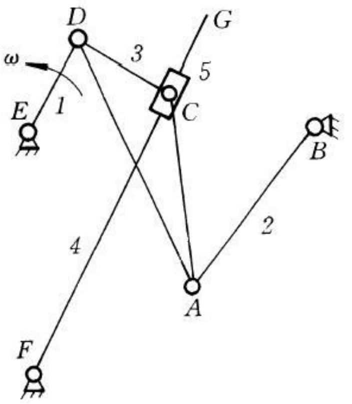


Fig. 2.57. Structural analysis of mechanism

This procedure is sometimes called group dividing. During group dividing, any link and kinematic pair can only belong to one group and cannot appear twice in different groups. According to the steps mentioned above, for the mechanism shown in Fig. 2.57, the assembly order of groups, type of group, link serial numbers, inner pair and outer pairs of each group are listed in Table 2.3. Since the highest grade of group in this mechanism is II, the mechanism is a grade II mechanism. During kinematic analysis of the mechanism, the first group must be analyzed first. Only after that, can the second group be analyzed.

Table 2.3. Structural analysis for the mechanism shown in Fig. 2.57.

	Type	Link serial numbers	Inner pair	Outer pairs
First group	*RRR*	2,3	*A*	*B,D*
Second group	*RPR*	4,5	sliding pair *C*4-5	*F*, revolute C3-5

The theory of structural analysis reveals the internal rule of the mechanism composition. It can help us to understand the structure of a mechanism, to analyze the transmission route in the mechanism, and to improve our ability in mechanism design.

2.17. Structural Synthesis

Whereas kinematic analysis aims at analyzing the motion inherent in a given machine or mechanism, kinematic synthesis aims at determining mechanisms that are required to fulfill certain motion specifications. Kinematic synthesis can, therefore, be thought of as a reverse problem to kinematic analysis of mechanisms. Synthesis is very fundamental of a design as it represents creation of a new hardware to meet particular needs of motion, namely displacement, velocity or acceleration-singly or in combination.

Probably, the most obvious external characteristics of a kinematic chain or mechanism are: the number of links and number of joints. Movability studies based on only these two parameters come under the name 'number synthesis.' The oldest and still the most useful (although with limitations) estimate of movability/mobility is known as the 'Grubler's criterion'. Effect of link lengths, directions and locations of axis, position of instantaneous centers of velocity, complexity of connections, etc. are neglected in this approach.

Mechanism number-synthesis is applied basically to linkages having turning pairs (pin joints) only. This does not, however, restrict its application to mechanisms with turning pairs alone. For it has been shown by that, having once developed complete variety of pin jointed mechanisms, the method can most readily be converted to accommodate cams, gears, belt drives, hydraulic cylinder mechanisms and clamping devices.

Following deductions will be useful in deriving possible link combinations of a given number of links for given degree of freedom. It is assumed that all joints are simple and there is no singular link.

From equation (2.1),

$$F = 3(n-1) - 2l .$$

Rewriting this equation, we have

$$l = \frac{3(n-1)}{2} - \frac{F}{2} . \tag{2.4}$$

Since total number of turning pairs must be an integer number, it follows that either $(n-1)$ and F should be both even or both odd. Thus, for l to be an integer number:

(1) If d.o.f. F is odd (say, 1,3,5…), $(n-1)$ should also be odd. In other words, n must be eve.

(2) If d.o.f. F is even (say 2,4…), $(n-1)$ should also be even. In other words for F to be even, n must be odd.

Summing up, for F to be even, n must be odd and for F to be odd, n must be even.

Let n_2 be a number of binary links, n_3 be a number of ternary links, n_4 - number of quaternary links, n_k - number of k-nary links.

The above number of links must add up to the total number of links in the mechanism.

Thus,

$$n = n_2 + n_3 + n_4 + ... + n_k \text{ or}$$

$$n = \sum_{i=2}^{k} n_i . \tag{2.5}$$

Since discussions are limited to simple jointed chains, each joint/pair consists of two elements. Thus, if e is total number of elements in the mechanism, then

$$e = 2l \,. \tag{2.6}$$

By definition binary, ternary, quaternary, etc. links consist of 2, 3, 4 elements respectively. Hence, total number of elements is also given by

$$e = 2n_2 + 3n_3 + 4n_4 + ... + k(n_k) \,. \tag{2.7}$$

Comparing right hand side of equations (2.6) and (2.7), we have

$$2l = 2n_2 + 3n_3 + 4n_4 + ... + k(n_k) \,. \tag{2.8}$$

Substituting (2.5) and (2.8) in (2.1), we have

$$F = 3[(n_2 + n_3 + n_4 + ... + n_k) - 1] - [2n_2 + 3n_3 + 4n_4 + ... + kn_k] \,.$$

Simplifying further,

$$F = [n_2 - n_4 - 2n_5 - 3n_6 - ... - (k-3)n_k] - 3 \text{ or rearranging,}$$

$$n_2 = (F+3) + [n_4 + 2n_5 + 3n_6 + ... + (k-3)n_k] \,. \tag{2.9}$$

Thus, number of binary links required in mechanism depends on d.o.f. and also on the number of links having elements greater than 3. Sacrificing exactness for fear of a complex relation, minimum number of binary links is deduced from eq. (2.9) as:

$$n_2 \geq 4, \text{ for } F = 1,$$

$$n_2 \geq 5, \text{ for } F = 2,$$

$$n_2 \geq 6 \text{ for } F = 3, \text{ etc.} \tag{2.10}$$

This proves that minimum number of binary links for F=1 is 4, while the minimum number of binary links required for F=2 is 5.

Let's determine maximum possible number of turning pairs on any of the n links in a mechanism

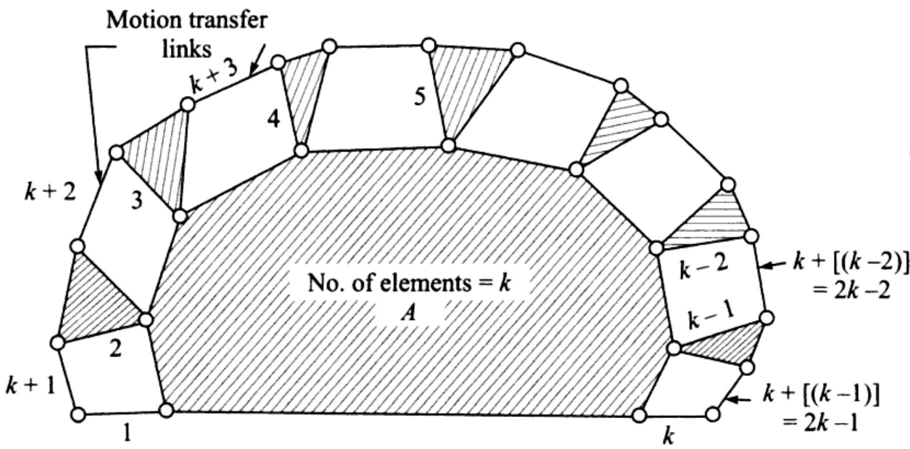


Fig. 2.58. Minimum number of link's required for closure

The problem is approached in an indirect manner. We pose the problem to be that of finding minimum number of links n required for closure when one of the links has largest number of elements equal k. An attempt is now made to close the chain in Fig.2.58 having link A of k elements.

For completing the chain with a minimum number of links involving no multiple joint, it is necessary to interconnect ternary links at all the elements of link A except the first and last element. Connecting ternaries at intermediate elements ensures a continuity of motion from link l to link k. Links directly connected to link A are labeled l rough k, while the motion transfer links shown in Fig.2.58 are numbered as $(k+1),(k+2),(k+3),...[k+(k-2)],[k+(k-1)]$. The last motion transfer link is thus numbered as $(2k-1)$. Clearly, minimum number of links required to complete the chain is $(2k-1)$, besides the link of highest elements.

In other words, for a given number of links $n=2k$, a link can have a maximum of k elements. Hence,

$$k=\frac{n}{2}. \tag{2.11}$$

Thus when n is even, maximum possible number of elements which a link can have is $n/2$.

An important conclusion emerging out of eq. (2.9) is that the number of ternary links does not have any influence on degrees of freedom of a mechanism.

For a mechanism with d.o.f. =1,

$$C=\frac{1}{2}(n)-1, \tag{2.12}$$

where C is a number of independent circuit or loops, n is a number of links.

2.18. Enumeration of kinematic Chains

Let N be the number of links and F degree of freedom, for which all possible planar chains are needed to be established.

Step 1. For given N and F establish the number of joints (hinges) using (2.4)

$$l=\frac{3N-(F+3)}{2}.$$

For F=1, this reduces to

$$l=\frac{3N-4}{2}.$$

Step 2. For given N, establish maximum number of elements permissible on any link, using

$$k=\frac{N}{2}, \text{for } F=1,3,5$$

$$\text{and } k=\frac{(N+1)}{2}, \text{for } F=2,4.$$

Step 3. Substituting expression for N and $2l$, namely,

$$N=n_2+n_3+n_4+...+n_k, \text{ and } 2l=2n_2+3n_3+4n_4+...+k(n_k)$$

in Grubler's eq. (2.1) we have

$$F=3[(n_2+n_3+n_4+...+n_k)-1]-[2n_2+3n_3+4n_4+...kn_k] \text{ or}$$

$$F=[n_2-(n_4+2n_5+...+(k-3)n_k)-3].$$

Thus for $F=1$,

$$n_2+n_4-2n_5+...+(k-3)n_k=4.$$

Above equations may be used to list all possible combinations of $n_2, n_4, n_5,...$ which satisfy given conditions.

Example 2.12. Enumerate all chains possible with $N=6$ and $F=1$.

Solution: Total number of hinges $l = \dfrac{3(6)-4}{2} = 7$.

Also, for even number of links $(N=6)$, maximum number of hinges on any link is 6/2=3. Thus the chains will consist of binary and ternary links only. Hence, we have from eqs. (2.5) and (2.8),

$$n_2 + n_3 = N = 6 \text{ and } 2n_2 + 3n_3 = 2l = 14. \tag{2.13}$$

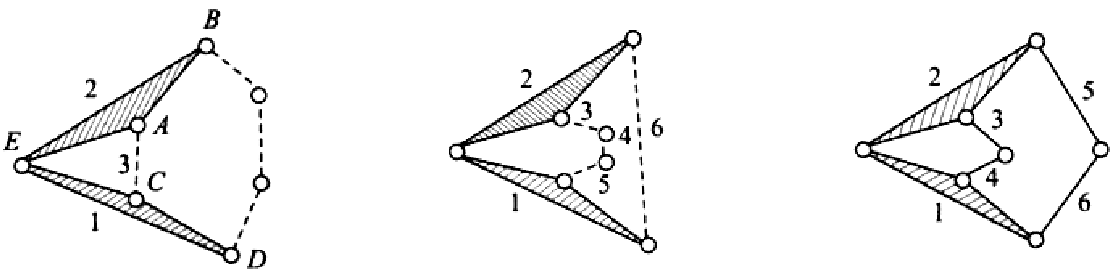
Substituting in Grubler's criterion we have,

$$3(n_2 + n_3) - (2n_2 + 3n_3) - 4 = 0 \text{ or } n_2 - 4 = 0.$$

Thus, $n_2 = 4$ and from eq. (2.13), $n_3 = 2$.

We begin by considering the ways in which links of highest degree (i.e., links having largest number of elements) can be interconnected. The two ternaries can be either connected directly through a common pair or can be connected only through one or two binary links.

In Fig.2.59 we consider the first possibility. The two ternaries of 2.59(a) and (b) cannot be connected through a single link as it amounts to forming a structural loop (3-link loop). The only way to connect them through 4 binaries (avoiding formation of a 3-link loop) is therefore, as shown at Figs.2.59(c), which gives Watt's



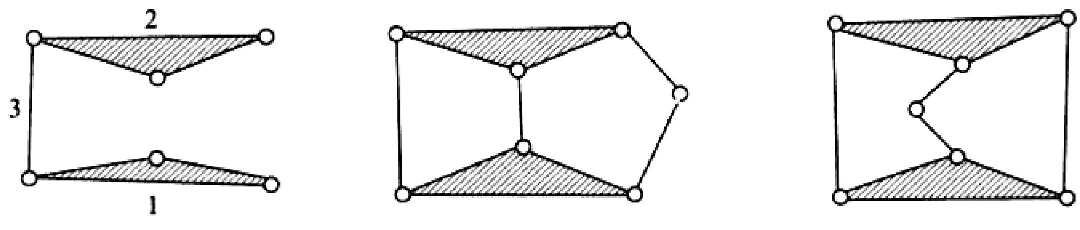
(a) Undesirable due to existence of 3-link loop 1-2-3; (b) Undesirable due to formation of link loop 1-2-6; (c) Permissible combination as no3-link loop exists

Fig. 2.59

chain.

Considering the second alternative, Fig. 2.60 (a) shows the two ternary links 1 and 2 being connected through a single binary link 3. Then, between one of remaining two pairs of elements of links 1 and 2, we may introduce a single binary and between the other pair of elements, two remaining binaries. The resulting arrangements are as at Fig.2.60 (b) and (c). It is easy to verify that arrangements at

Figs. 2.60 (b) and (c) are structurally the same.



(b) Valid Stephenson's chain; (c) Valid chain (same as that at b)
Fig. 2.60

Example 2.13. Enumerate all possible chains of $N = 7$ and F =2.

Solution: Total number of hinges $l = \frac{3(N)-(F+3)}{2} = \frac{3(7)-(2+3)}{2} = 8$

Maximum number of elements on any link: $\leq (N+1)/2 = 4$.

Hence, only binary, ternary and quaternary links are possible. Thus, from eqs. (2.5) and (2.8),

$$n_2 + n_3 + n_4 = N(=7),\ 2n_2 + 3n_3 + 4n_4 = 2l(=16).$$

Substituting in Grubler’s equation,

$$F = 3(N-1) - 2l,$$

we have

$$2 = 3[(n_2 + n_3 + n_4) - 1] - (2n_2 + 3n_3 + 4n_4) \text{ or } n_2 - n_4 = 5.$$

Thus, the possible combinations are (Note that for any mechanism with $F = 2, n_2 \geq 5$): $n_4 = 1$; $n_2 = 6$; $n_4 = 0$; $n_2 = 5$. Also check that $n_2 + n_3 + n_4 = 7$.

Obviously, remaining links in above combinations will be the ternaries. Thus the two combinations possible are:

n_4	n_3	n_2	Total N
1	0	6	7
0	2	5	7

Different chains that can be formed are as shown in Figs.2.61 (a), (b), (c) and (d). Chain at Fig.2.61 (a) involves a quaternary link with remaining 6 binary links forming two independent loops of *F*= 1. A mechanism of *F*=2 is possible only when any link other than quaternary is fixed. Chain at Fig. 2.61 (b) involves two ternary links that are directly connected. A binary cannot be used singly to connect these ternaries at any of the remaining pairs of elements as that leads to a 3-link loop. Therefore, the only option is to connect these ternaries through two binaries and through three binaries at remaining pairs of elements. This is shown in Fig. 2.61 (b). When the two ternaries are connected through a single binary, the two possible ways of incorporating remaining 4 binaries are shown at Fig. 2.61 (c) and (d).

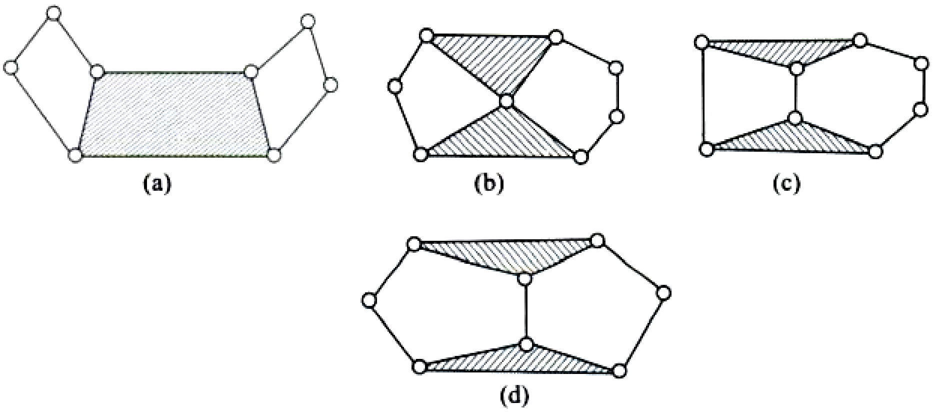


Fig. 2.61. Feasible chains of seven links

Example 2.14. Enumerate combination of links possible in case of a 8-link chain with $F = 1$.

Solution: Number, of hinges $l = \dfrac{3(8)-4}{2} = 10$.

Maximum number of elements on one link is $\dfrac{8}{2} = 4$.

Hence the chains can have binary, ternary and quaternary links only. From equations (2.5) and (2.8),

$$n_2 + n_3 + n_4 = 8 \text{ and } 2n_2 + 3n_3 + 4n_4 = 20.$$

Substituting in Grubler's criterion,

$$3(n_2 + n_3 + n_4) - (2n_2 + 3n_3 + 4n_4) - 4 = 0 or (n_2 - n_4) = 4.$$

We study various combinations indicated by above expression in respect of their viability:

n_4 (assumed)	$n_2 = n_4 + 4$	$n_3 = 8 - (n_2 + n_4)$	Remark
4	8	-	Not acceptable as $n_2 + n_4 > N$
3	7	-	Not acceptable as $n_2 + n_4 > N$
2	6	-	acceptable
1	5	2	acceptable
0	4	4	acceptable

Thus the three valid combinations of links are:

(1) $n_4 = 2$; $n_3 = 0$; $n_2 = 6$;

(2) $n_4 = 1$; $n_3 = 2$; $n_2 = 5$;

(3) $n_4 = 0$; $n_3 = 4$; $n_2 = 4$.

The first combination yields the following two chains ($n_4 = 2; n_3 = 0; n_2 = 6$). (Fig. 2.62).

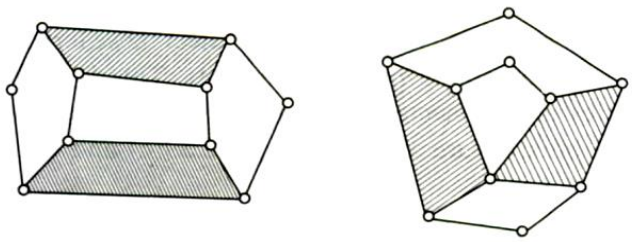


Fig. 2.62. Feasible chains for first combination

The second combination of

links yields the following five chains ($n_4 = 1; n_3 = 2; n_2 = 5$) (*see* Fig. 2.63).

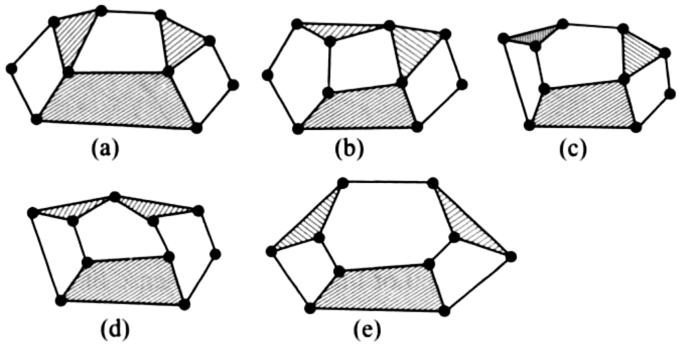


Fig. 2.63. Feasible chains for second combination

The third combination ($n_4 = 0; n_3 = 4; n_2 = 4$) of links yields following chains (see Fig 2.64)

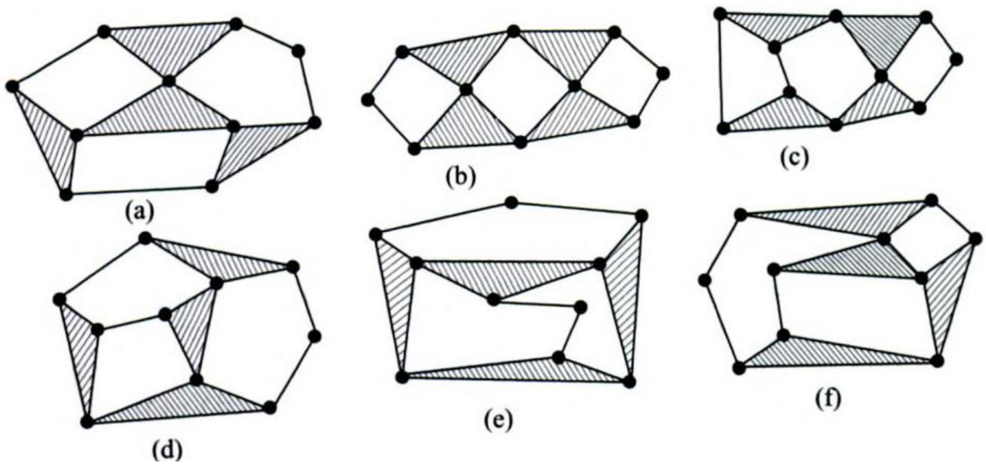


Fig. 2.64. Feasible chains for third combination